



ON Semiconductor®

Electronics System Thermal Design and Characterization

Roger Stout, P.E.

Senior Research Scientist

Corporate Research & Development

Advanced Packaging Technology

<roger.stout@onsemi.com>



Course outline

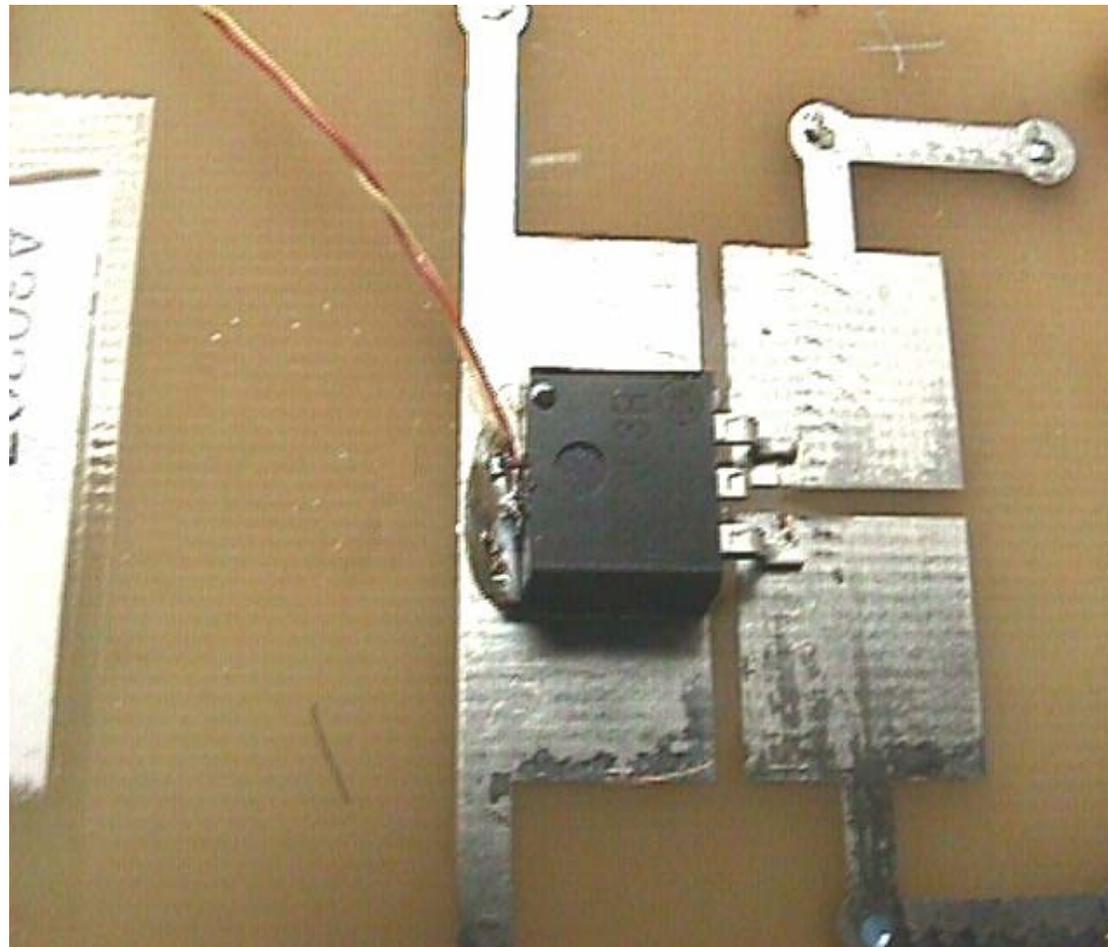
- Introduction
- Experimental Techniques
- Linear Superposition
- Thermal Runaway
- References



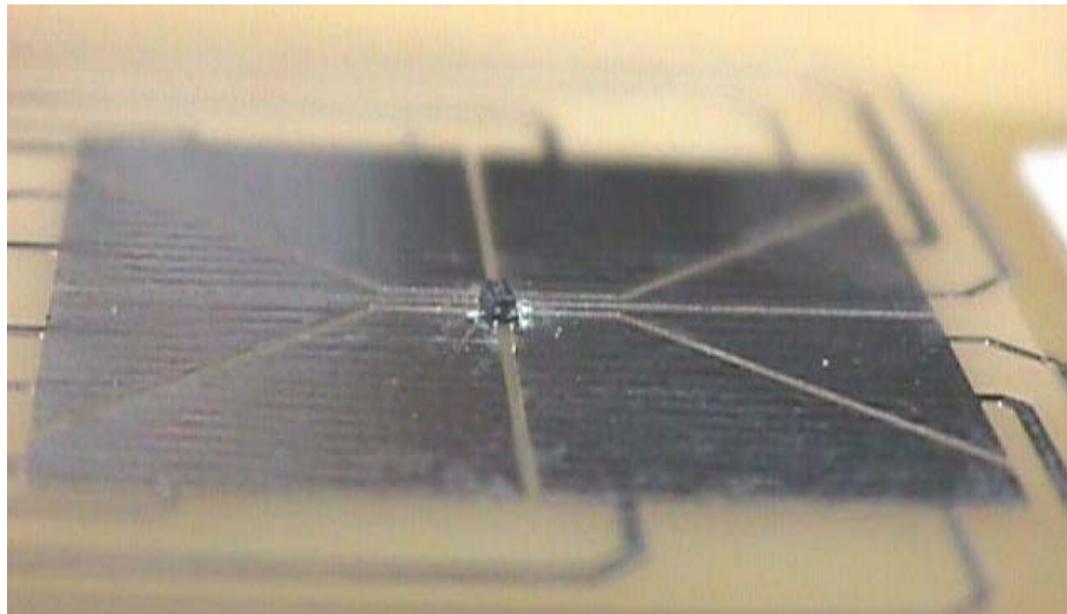
Introduction

- Why This Course?
- Terminology and Basic Principles
- Facts and Fallacies

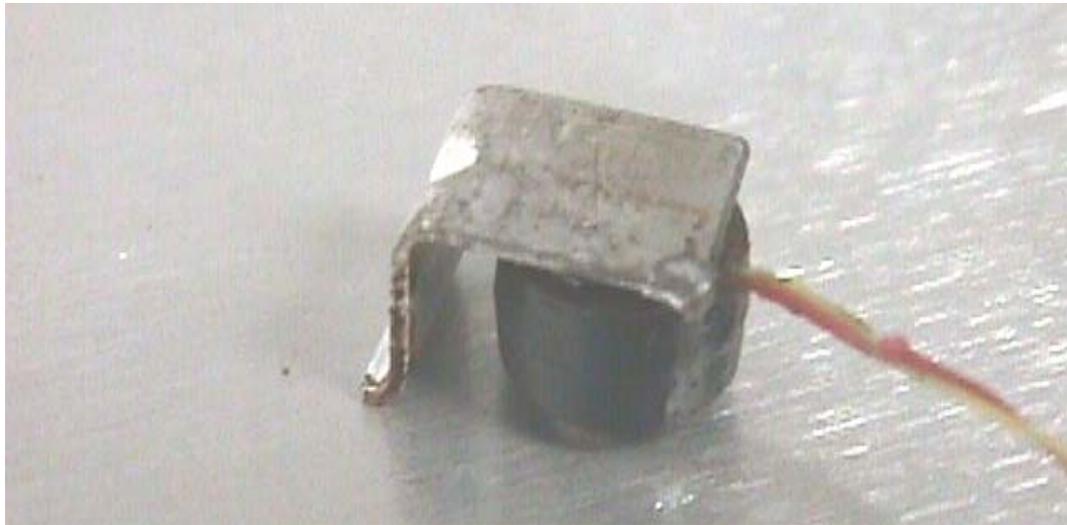
Can this device handle 2W?



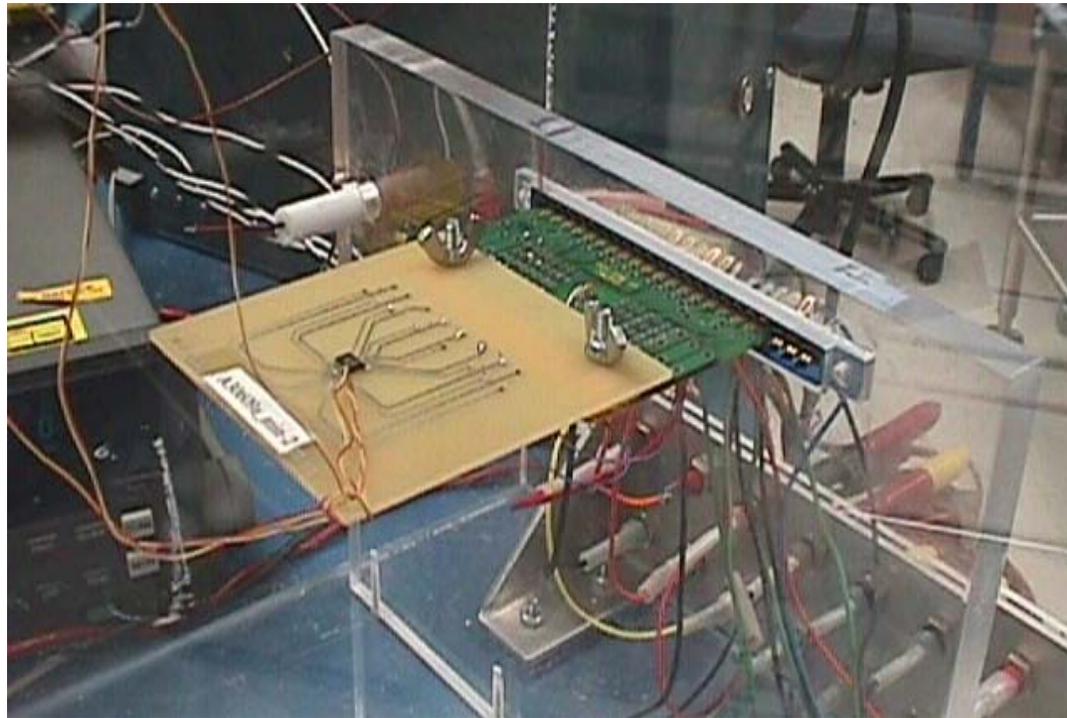
I'm putting 5A into this part. What's its junction temperature going to be?



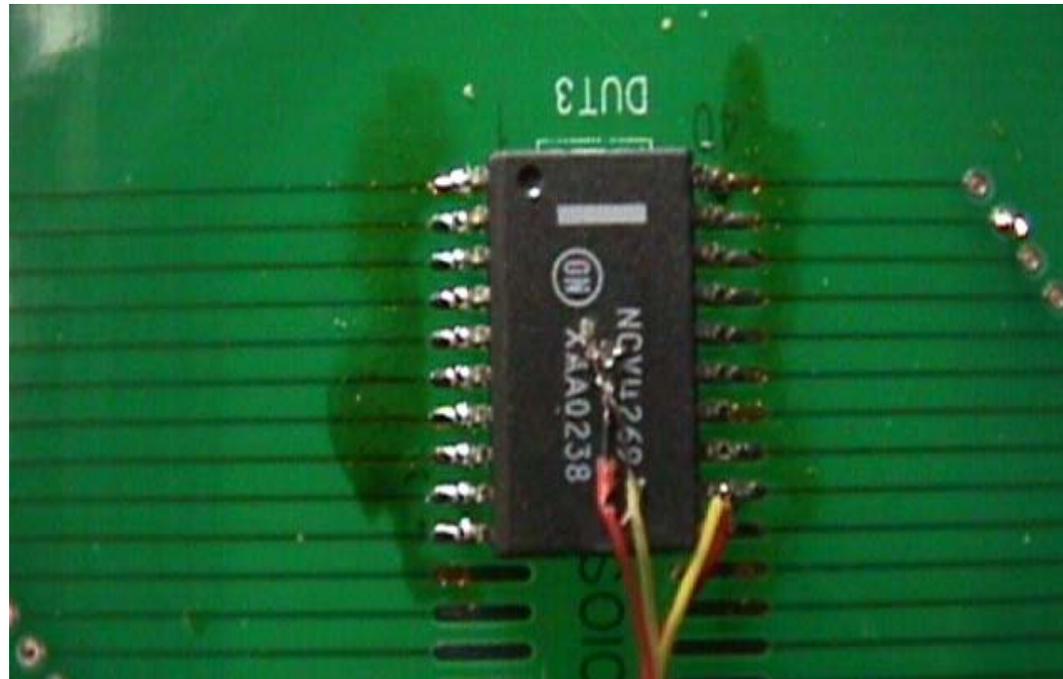
**I'm putting a 60W, 800ns pulse into this rectifier.
How much copper area do I need to make this
part work in my system?**



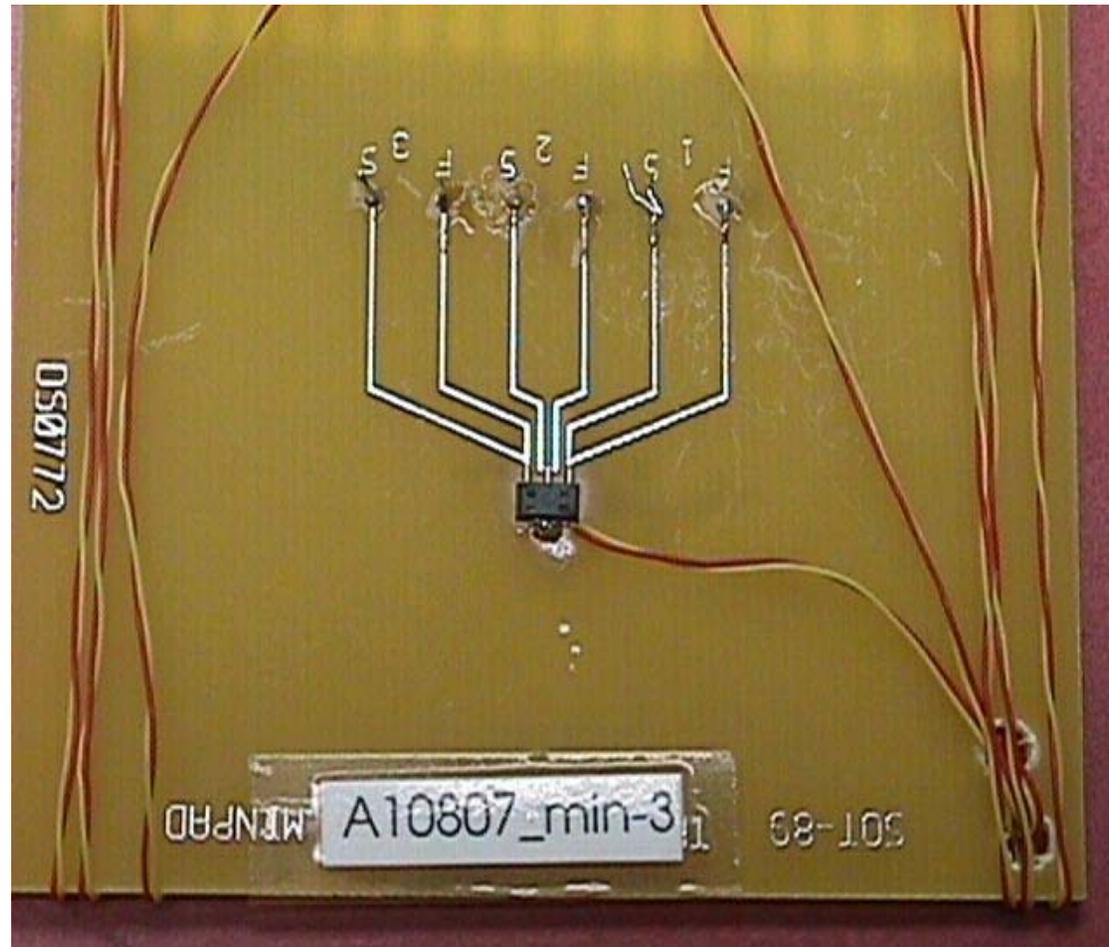
I'm putting together a data sheet for this new device. What's theta-JA for this package?



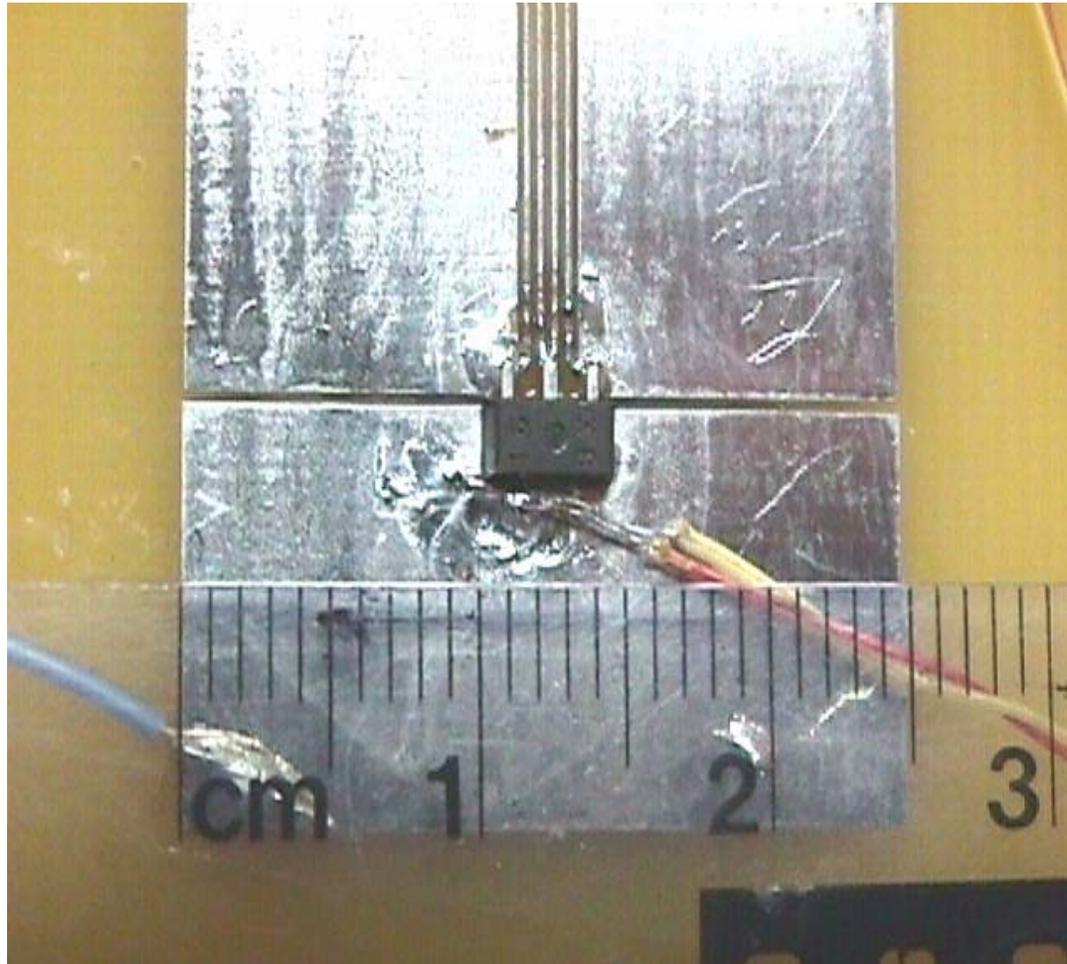
I'm putting together a data sheet for this new device. What's theta-JA for this package?



What's the *maximum power* rating on this part going to be?



What's the maximum power rating on *this* part going to be?



Why is our SOT-23 thermal number so much worse than our competition?

- Us
 - SOT-23 package
 - 60x60 die
 - solder D/A
 - copper leadframe
 - min-pad board
 - still air
- Them
 - SOT-23 package
 - 20x20 die
 - epoxy D/A
 - alloy 42 leadframe
 - 1" x 2oz spreader
 - big fan

Why θ_{JA} doesn't belong in the “Maximum Ratings”* table

*let alone the “Absolute Maximum Ratings”

It's like trying to sell your car (some bureaucrat says you *must* list its gas mileage in the ad)

For sale:

Geo Metro, 1999 model, excellent condition!

MAXIMUM RATINGS

Description	Symbol	Value	Units
Gas Mileage (Note 1)		4	mpg

¹ 20% grade uphill, 75mph, back seat and trunk full of bricks

Gee, we'd better not be so "worst case," should we?

For sale:

Geo Metro, 1999 model, excellent condition!

MAXIMUM RATINGS

Description	Symbol	Value	Units
Gas Mileage (Note 1)		10	mpg
Mileage derating factor		0.002	mpg/brick

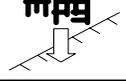
¹ 20% grade uphill, 75mph

Wait, they said “*maximum*”. Maybe we’re thinking about this all wrong ...

For sale:

Geo Metro, 1999 model, excellent condition!

MAXIMUM RATINGS

Description	Symbol	Value	Units
Gas Mileage (Note 1)		110	mpg
BDF (brick derating factor)		0.002	mpg/brick
IDF (incline derating factor)		2	mpg/%
SDF (speed derating factor)		0.07	mpg/mph

¹ 20% grade downhill, empty vehicle (no bricks, not even a driver!), coasting

Frankly, T_{j-max} is the only
“thermal” specification that I
think belongs in the Maximum
Ratings table.



Terminology and basic principles

“Junction” temperature?

Historically, for discrete devices, the “junction” was literally the essential “pn” junction of the device. This is still true for basic rectifiers, bipolar transistors, and many other devices.

More generally, however, by “junction” these days we mean the hottest place in the device, which will be somewhere on the silicon (2nd Law of Thermodynamics).

This gets to be somewhat tricky to identify as we move to complex devices where different parts of the silicon do different jobs at different times.

Thermal/electrical analogy

temperature \Leftrightarrow voltage

power \Leftrightarrow current

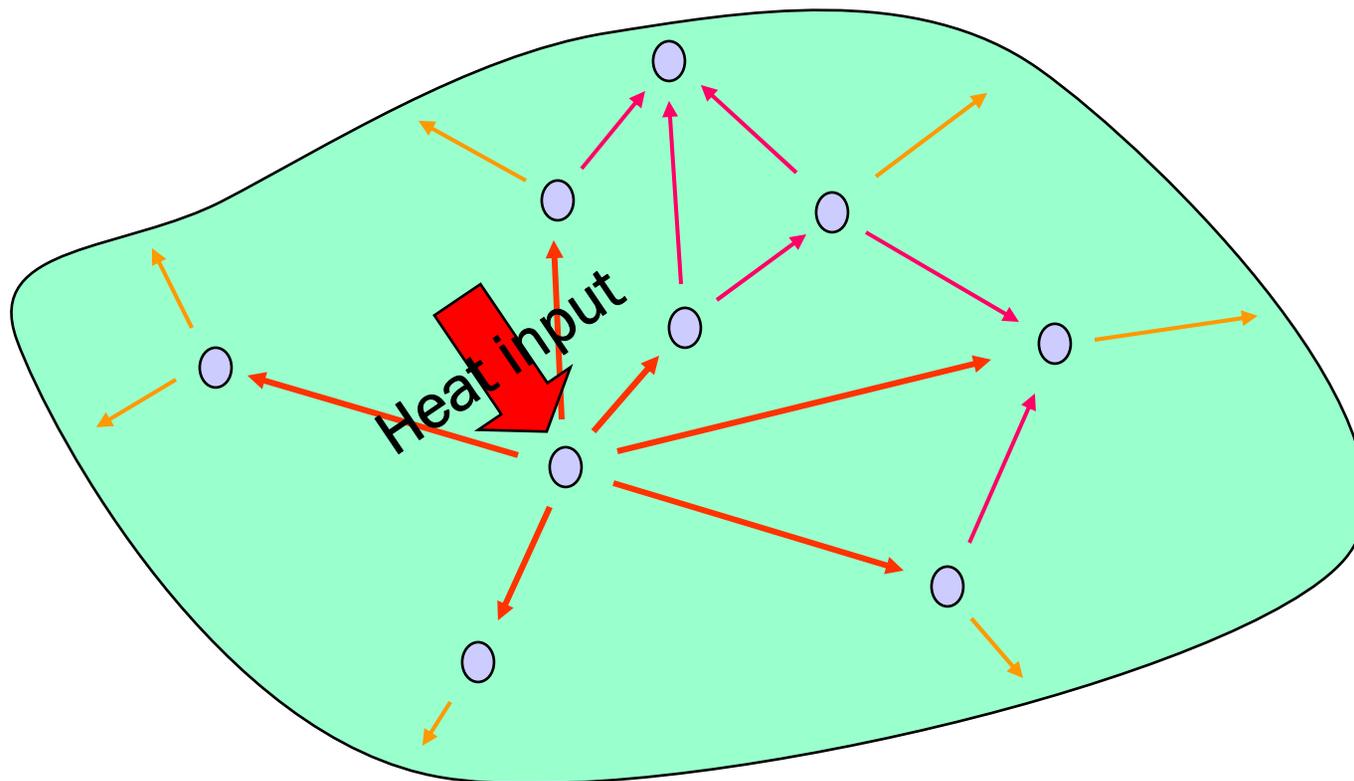
Δ temp/power \Leftrightarrow resistance

energy/degree \Leftrightarrow capacitance

Theta (θ) vs. psi (Ψ)

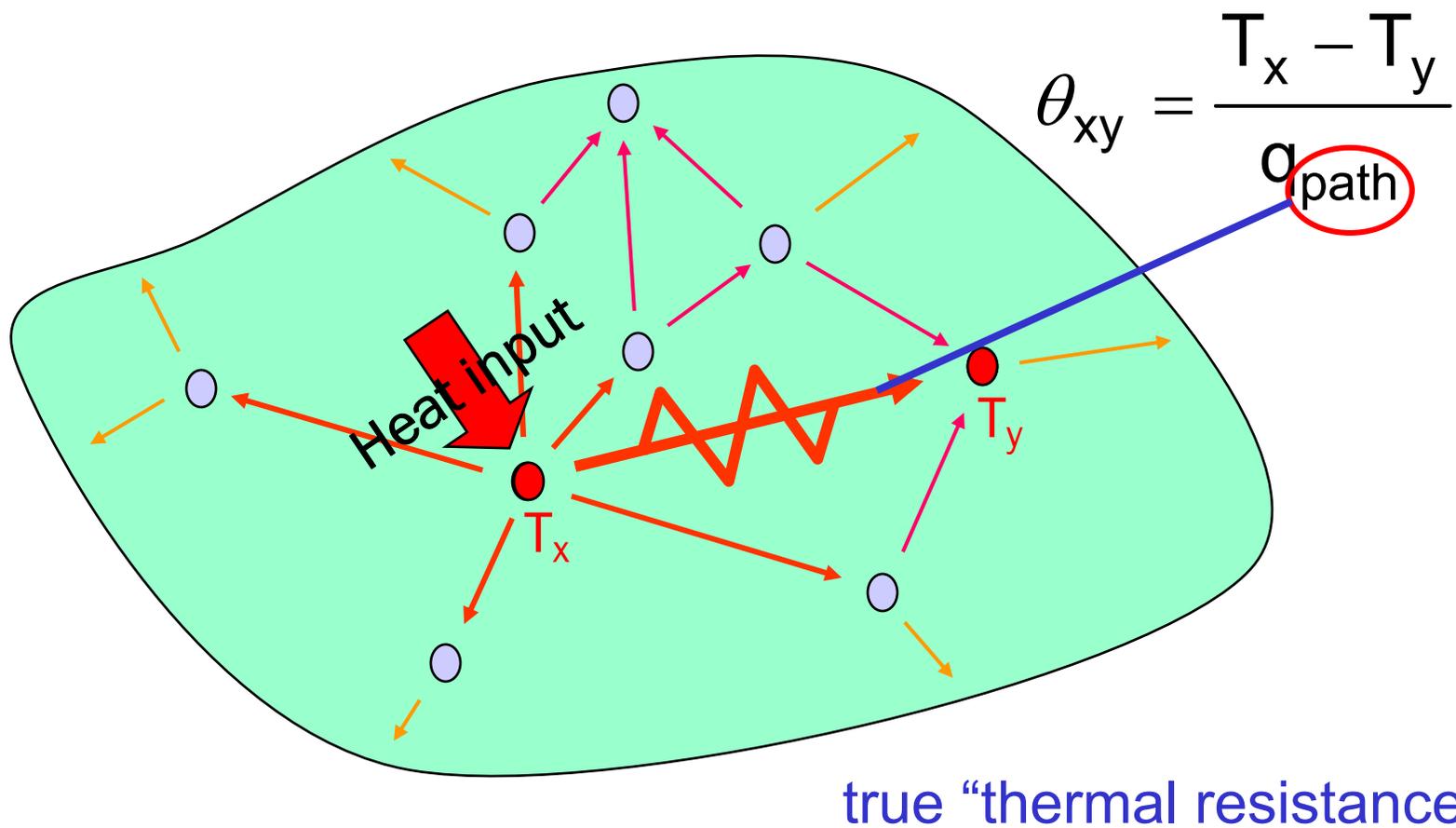
- JEDEC <<http://www.jedec.org/>> terminology
 - $Z_{\theta JX}$, $R_{\theta JA}$ older terms ref JESD23-3, 23-4
 - θ_{JA} ref JESD 51, 51-1
 - θ_{JMA} ref JESD 51-6
 - Ψ_{JT} , Ψ_{TA} ref JESD 51-2
 - Ψ_{JB} , Ψ_{BA} ref JESD 51-6, 51-8
 - $R_{\theta JB}$ ref JESD 51-8
 - **Great overview, all terms: JESD 51-12**

A generic thermal system



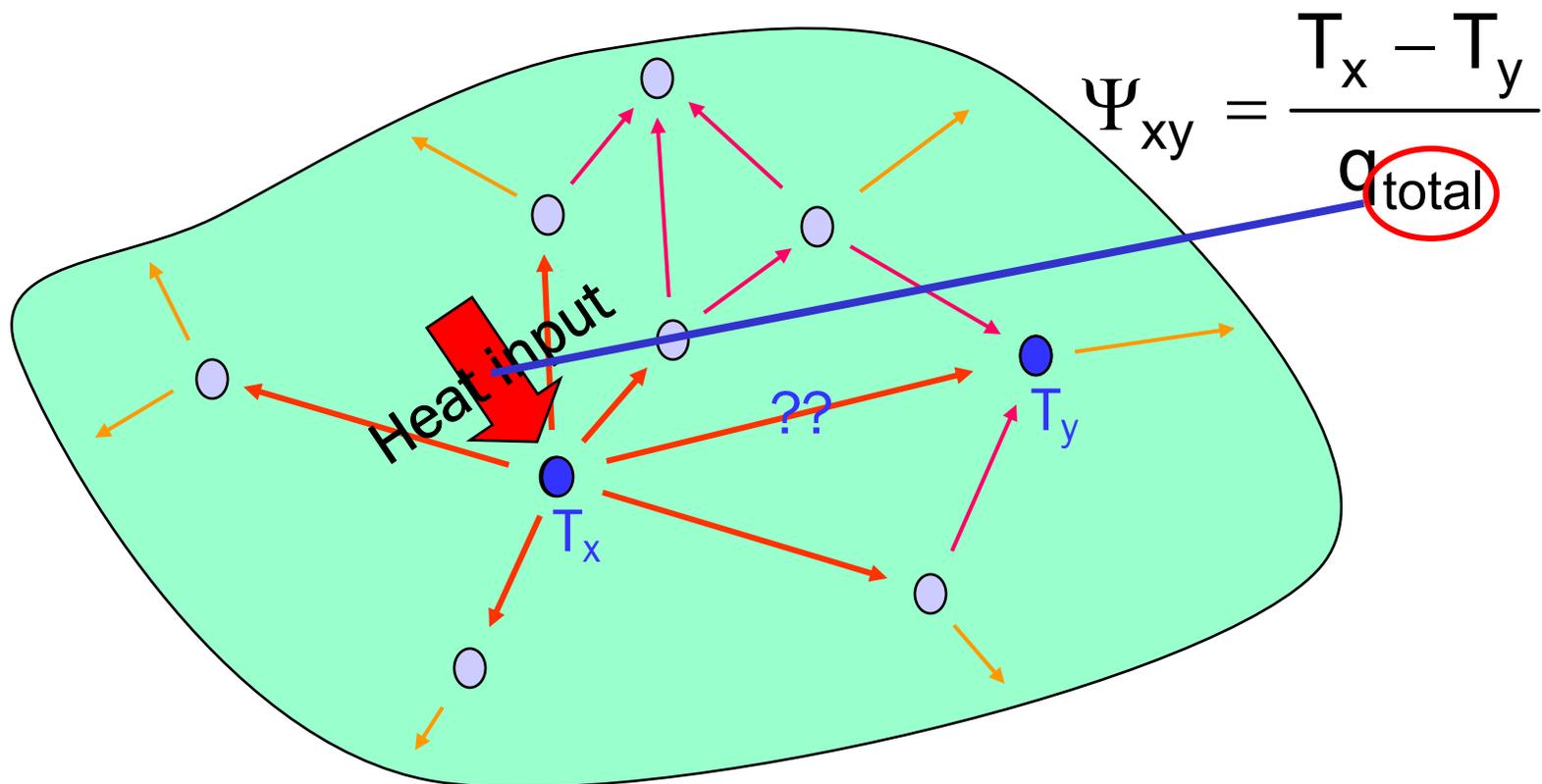
“Theta” (Greek letter θ)

We know actual heat flowing along path of interest



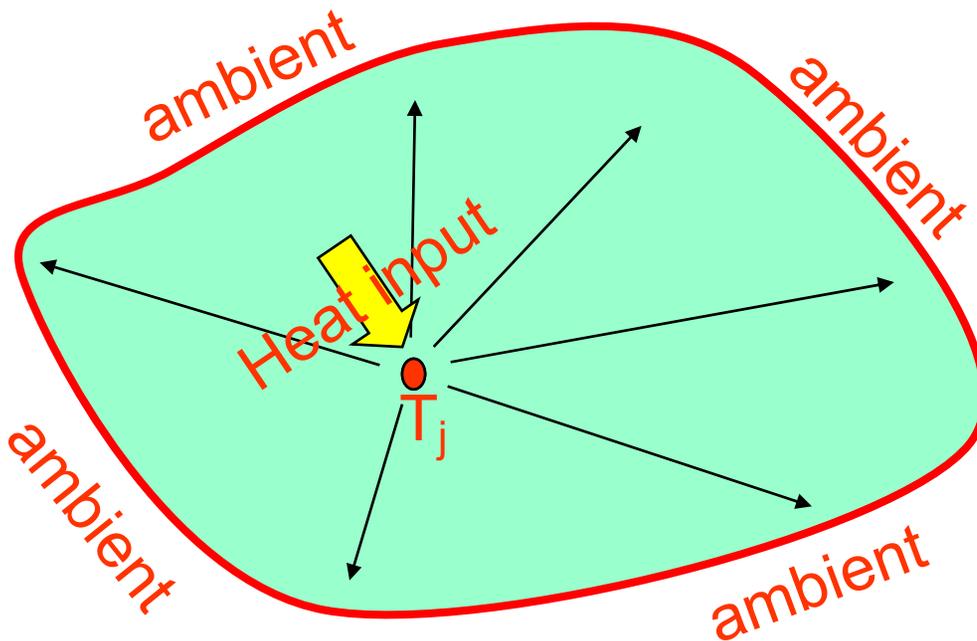
“Psi” (Greek letter Ψ)

We don't know actual heat flowing along path of interest



All we know is total heat input

When Ψ becomes θ



Either or both “points”
of interest are
isotherms

$$T_x = T_j \quad (\text{a point})$$

$$T_y = \text{ambient} \quad (\text{an isotherm})$$

All heat flowing
between them is
known

$$Power_{\text{path}} = Power_{\text{device}}$$

$$\theta_{JA} = \frac{T_j - T_{\text{ambient}}}{Power_{\text{total}}} = \Psi_{xy}$$

An example of a device with two different “Max Power” ratings

- **Suppose a datasheet says:**

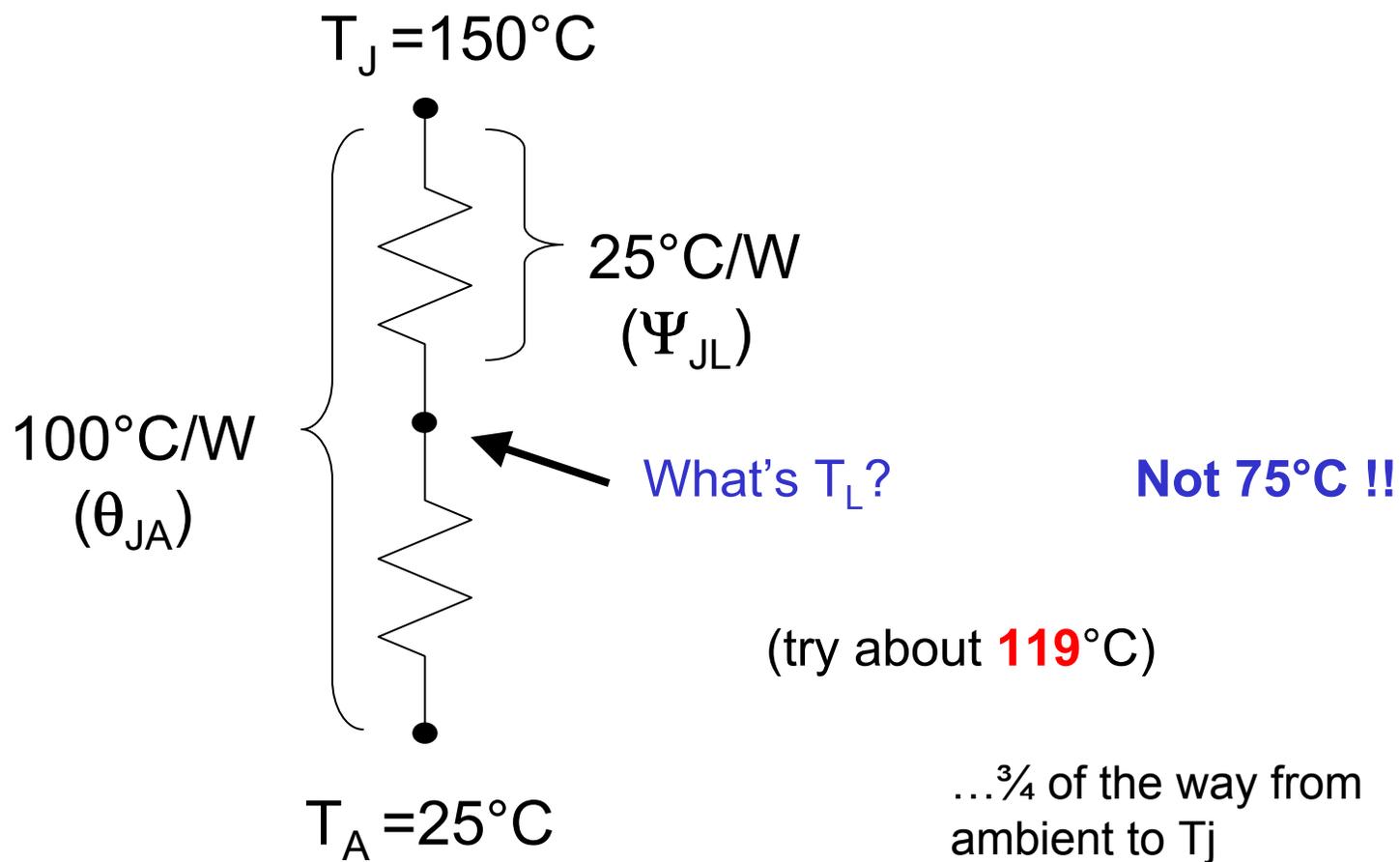
- $T_{jmax} = 150^{\circ}\text{C}$
 - $\theta_{JA} = 100^{\circ}\text{C/W}$
 - $P_d = 1.25\text{W}$ ($T_{amb} = 25^{\circ}\text{C}$)
- $$25 + 100 * 1.25$$
- $$= 25 + 125 = 150$$

- **But it also says:**

- $\Psi_{JL} = 25^{\circ}\text{C/W}$
 - $P_d = 3.0\text{W}$ ($T_L = 75^{\circ}\text{C}$)
- $$75 + 25 * 3$$
- $$= 75 + 75 = 150$$

Where’s the “inconsistency”?

Where's the inconsistency?





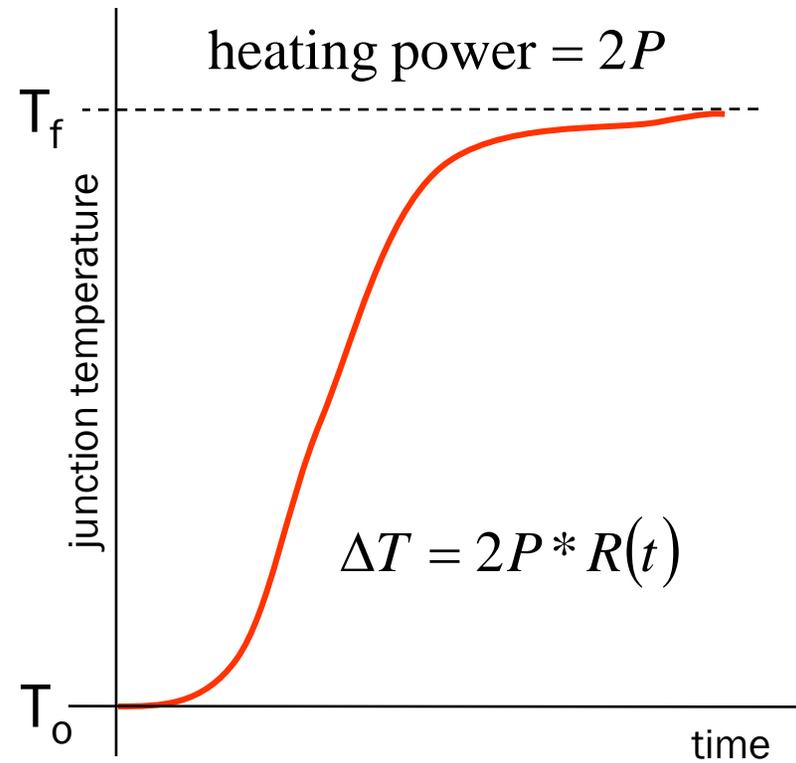
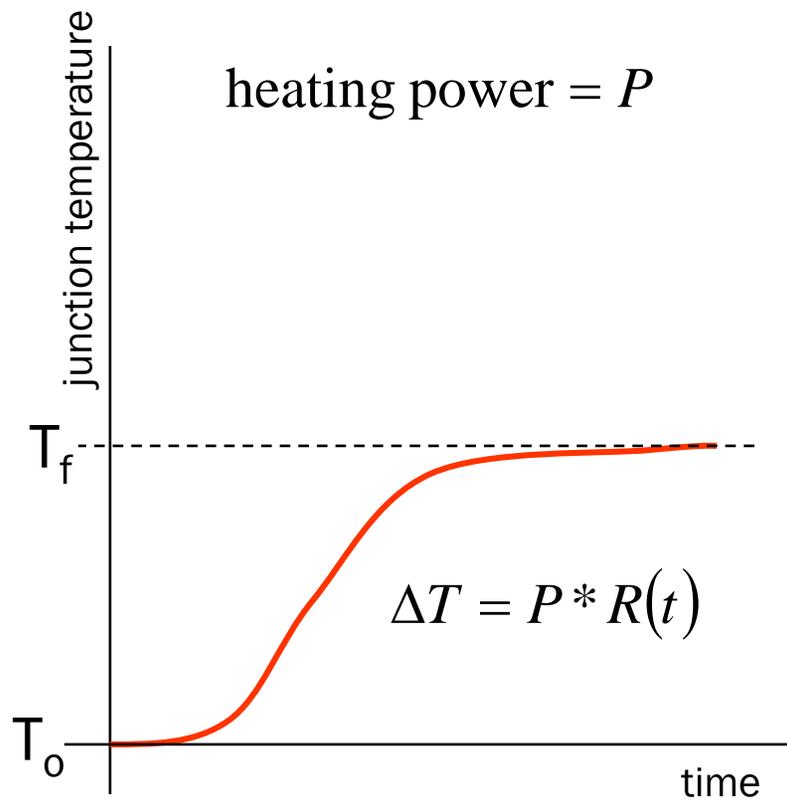
Facts and fallacies

Facts and fallacies

- Basic idea:
 - temperature difference is proportional to heat input

$$\Delta T \propto \text{Power}$$

twice the heat, twice the temperature rise



Facts and fallacies

- Basic idea:
 - temperature difference is proportional to heat input
- There are three modes of heat transfer
 - conduction
 - convection
 - radiation (electromagnetic/infrared)

Facts and fallacies

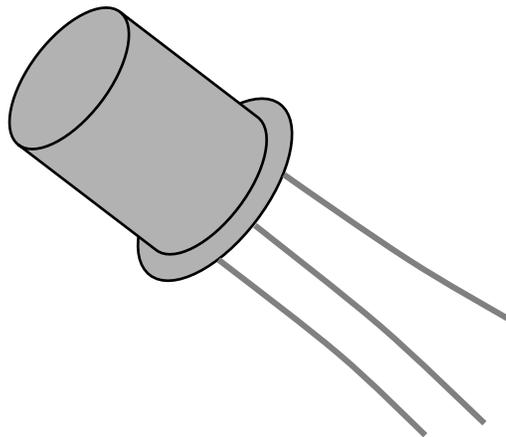
- Basic idea:
 - temperature difference is proportional to heat input
- Flaws in idea:
 - conduction effects (material properties)
 - depend on temperature
 - convection effects (esp. “still air”)
 - depend on temperature
 - radiation effects
 - depend on temperature

Facts and fallacies, cont'

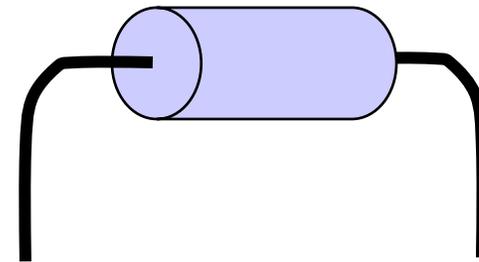
- Basic idea:
 - “thermal resistance” is an intrinsic property of a package

back in the good old days ...

metal can --
fair approximation of
“isothermal” surface



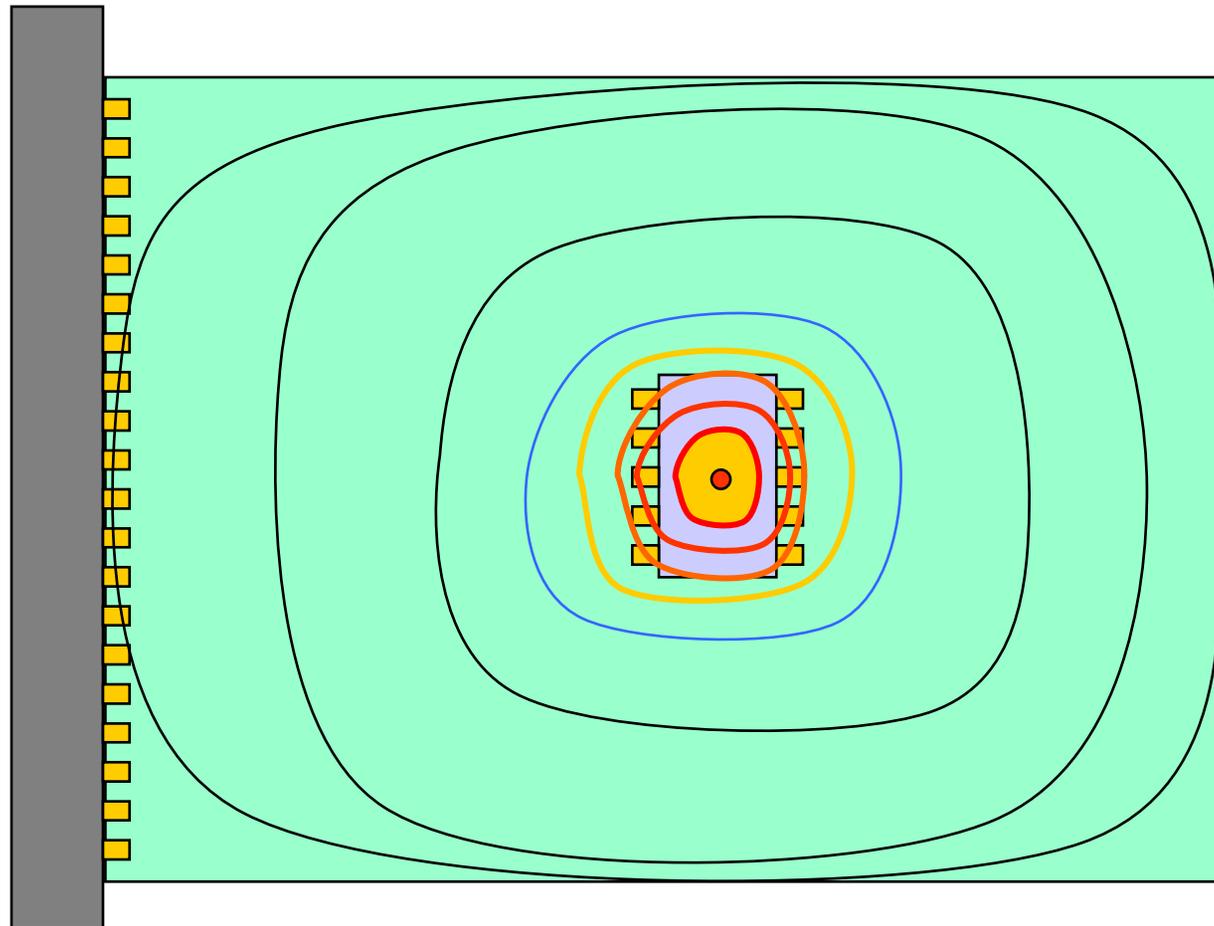
axial leaded device --
only two leads, heat
path fairly well defined



Facts and fallacies, cont'

- Basic idea:
 - “thermal resistance” is an intrinsic property of a package
- Flaws in idea:
 - there is no isothermal “surface”, so you can’t define a “case” temperature
 - Plastic body (especially) has big gradients
 - different leads are at different temperatures

Which lead? Where on case?



Facts and fallacies, cont'

- Basic idea:
 - “thermal resistance” is an intrinsic property of a package
- Flaws in idea:
 - there is no isothermal “surface”, so you can’t define a “case” temperature
 - Plastic body (especially) has big gradients
 - different leads are at different temperatures
 - **multiple, parallel thermal paths out of package**

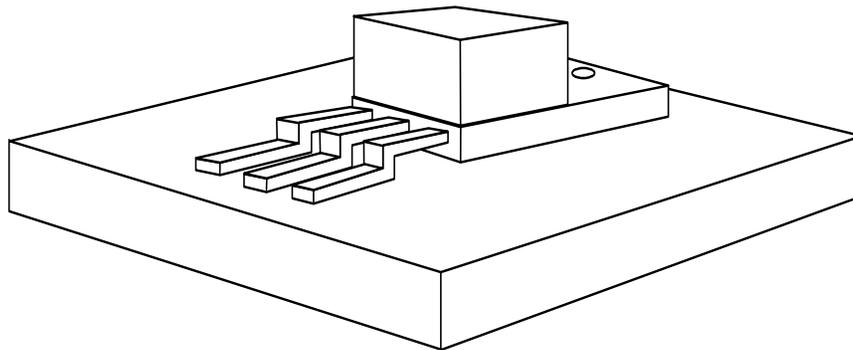
Same ref, different values

$$\Psi_{J-tab} = 1.2^{\circ}\text{C/W}$$

$$P_d = 50\text{W}$$

$$T_c = 25^{\circ}\text{C}$$

1 GPM of H₂O

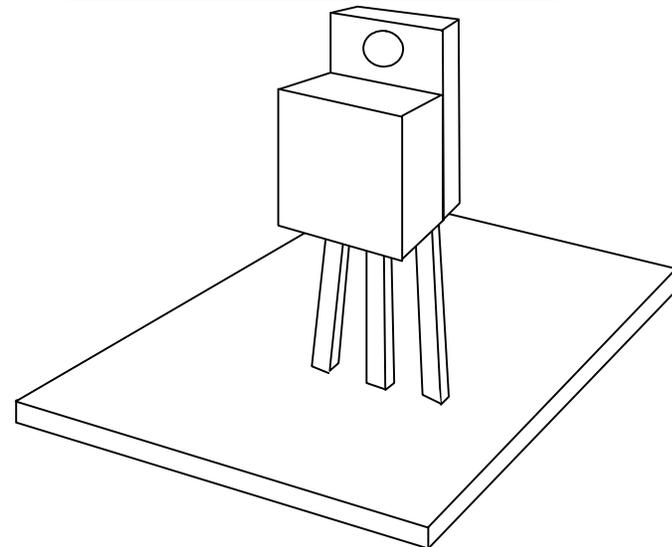


$$\Psi_{J-tab} = 0.8^{\circ}\text{C/W}$$

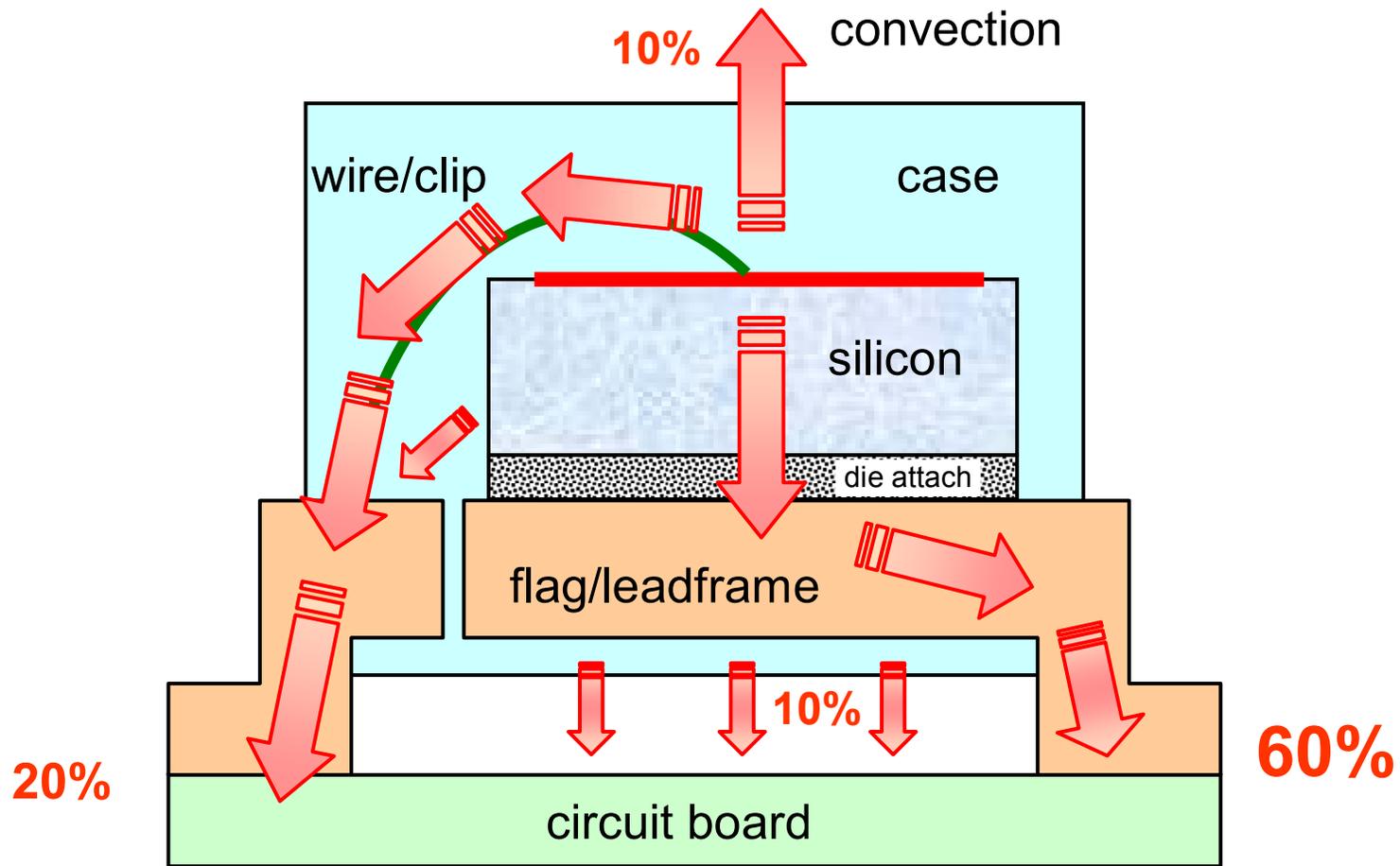
$$P_d = 1.5\text{W}$$

$$T_c = 25^{\circ}\text{C}$$

still air

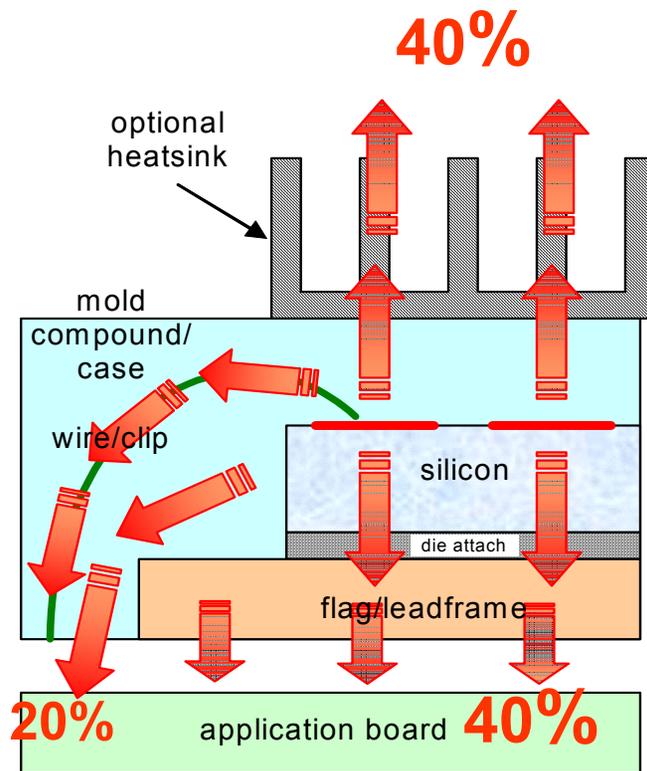


Archetypal package

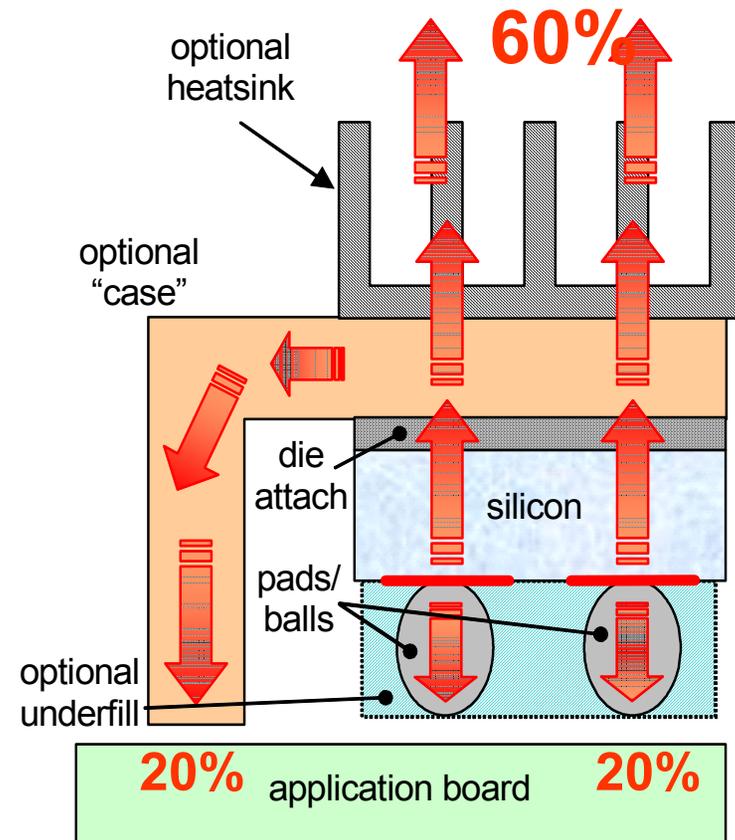


Then we change things ...

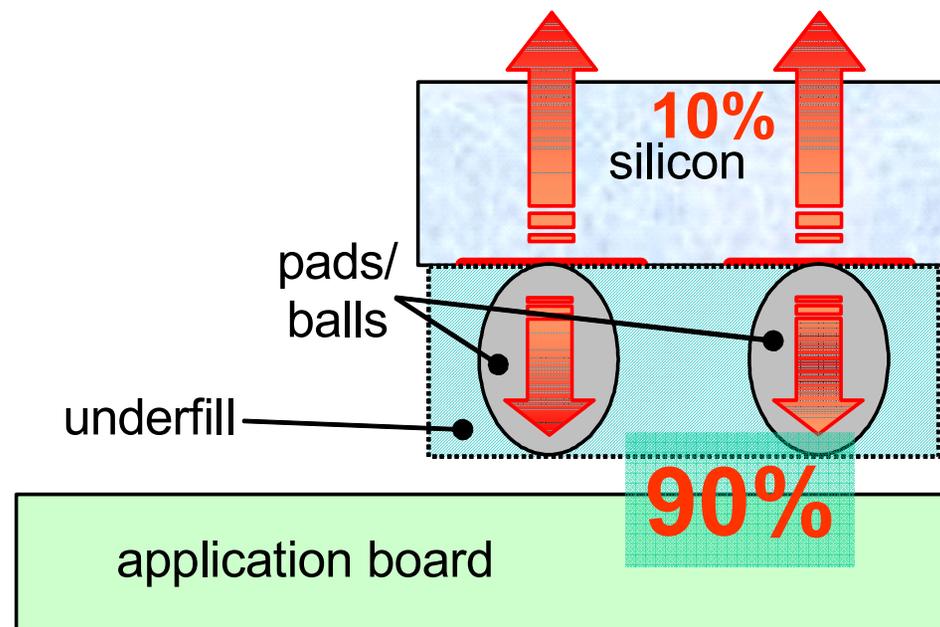
add an external heatsink ...



flip the die over ...

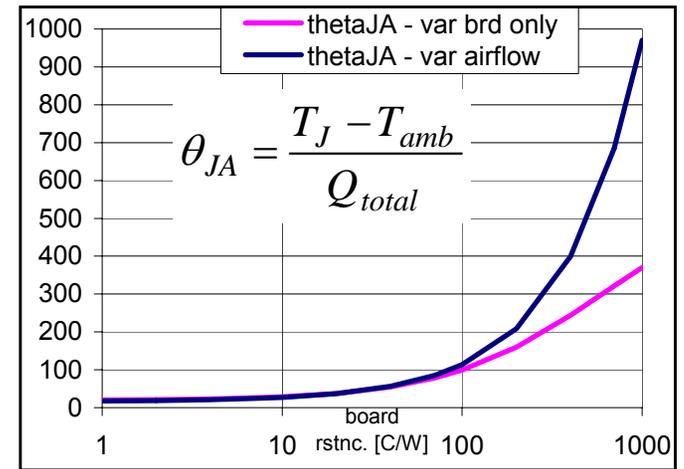
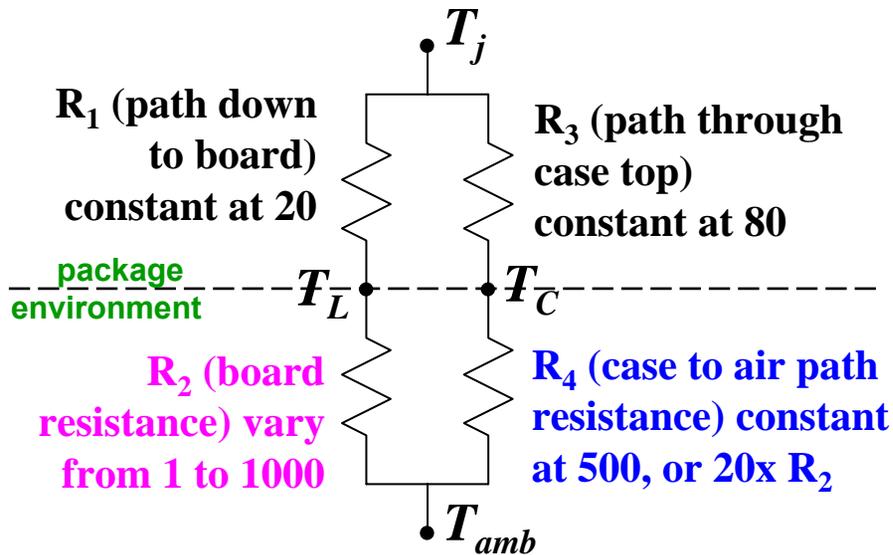


A bare “flip chip”





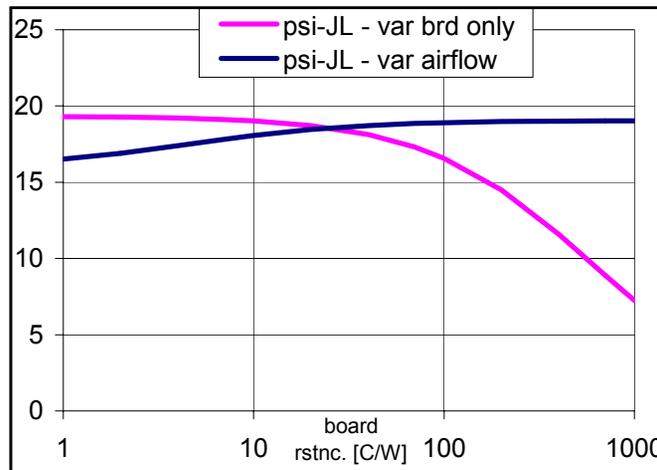
Even when it's constant, it's not!



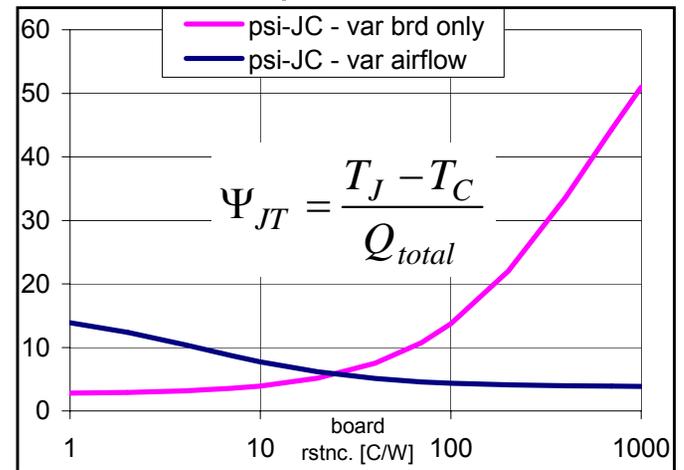
theta-JA

psi-JL

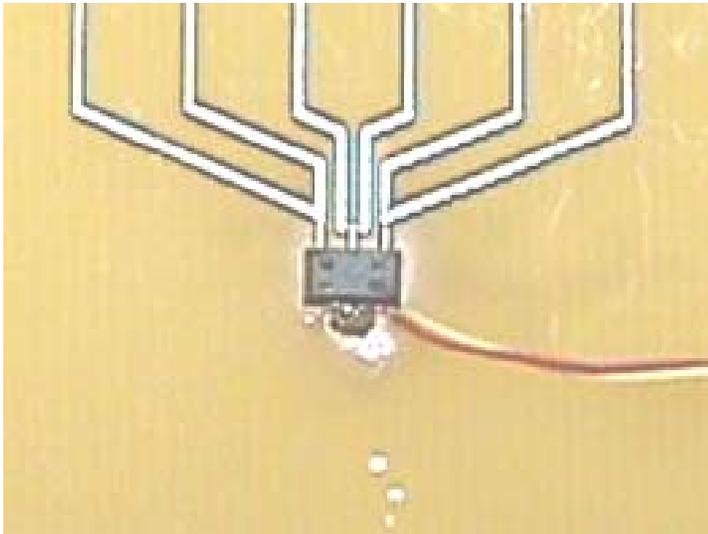
$$\Psi_{JL} = \frac{T_J - T_L}{Q_{total}}$$



psi-JT

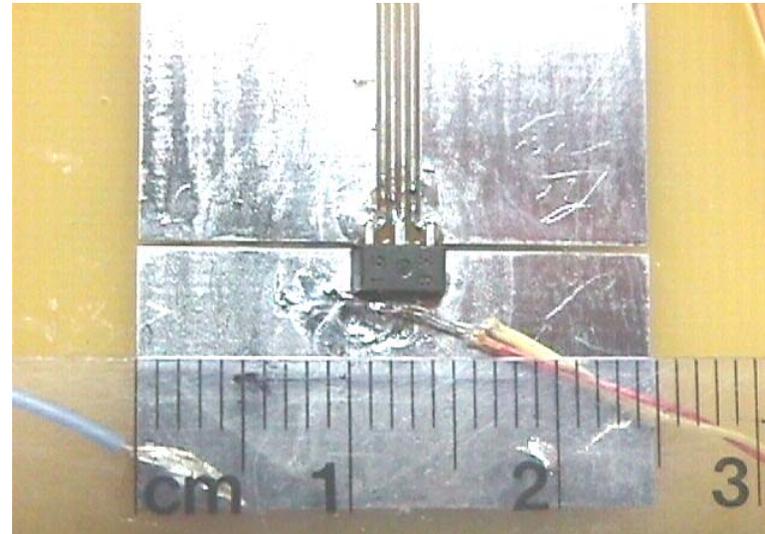


Typical thermal test board types



min-pad board

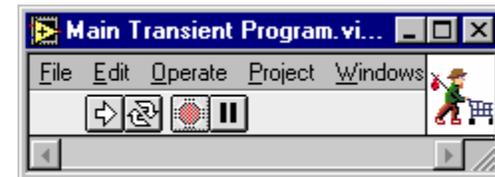
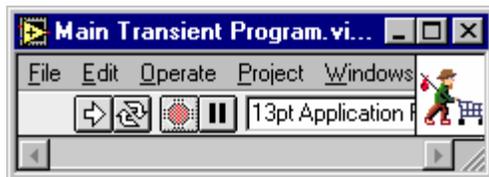
minimum metal area to attach device (plus traces to get signals and power in and out)



1-inch-pad board

device at center of 1" x 1" metal area (typically 1-oz Cu);
divided into sections based on lead count

Experimental Techniques



Experimental Techniques

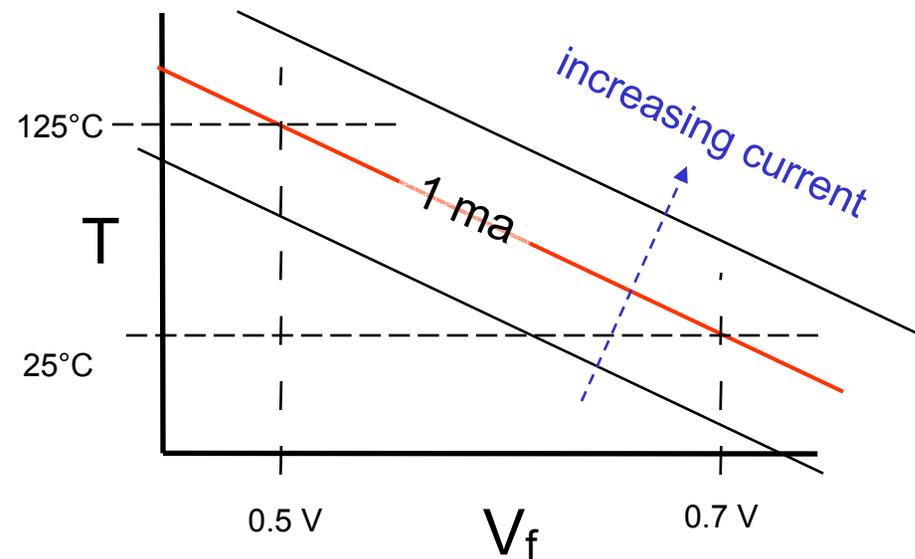
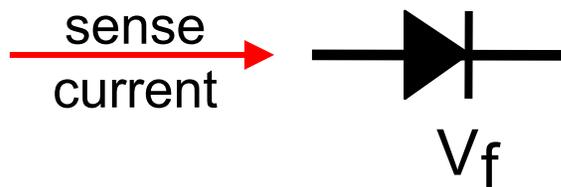
- Temperature Sensitive Parameters (TSPs)
- Different Device Types and How to Test Them
- Heating vs Cooling Curve Techniques
- Test Conditions

Temperature Sensitive Parameters

- JEDEC 51-1 good synopsis
- Basic diode physics (pg 5 of JESD 51-1)
 - At constant current, forward voltage goes down (linearly) with increasing temperature
- In principle, any device which has repeatable (not necessarily linear) voltage vs. temperature characteristics can be used
- Commercial thermal test equipment typically requires linear TSP behavior

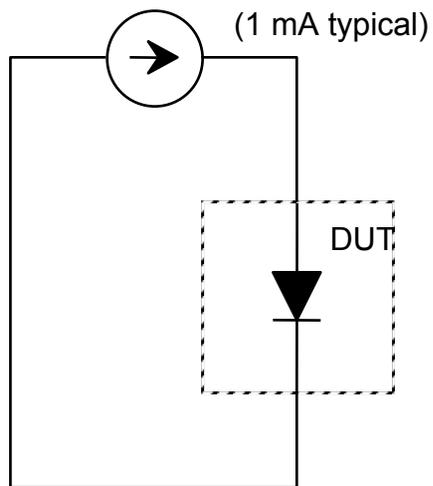
Typical TSP Behavior

calibrate forward voltage at controlled,
small (say 1mA) sense current



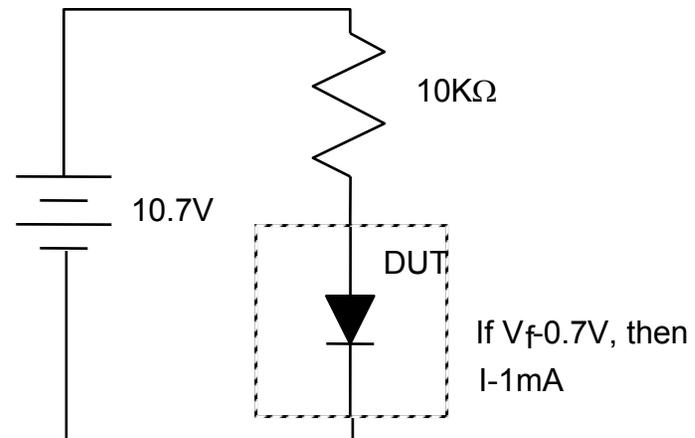
How to measure T_j

true const. current supply



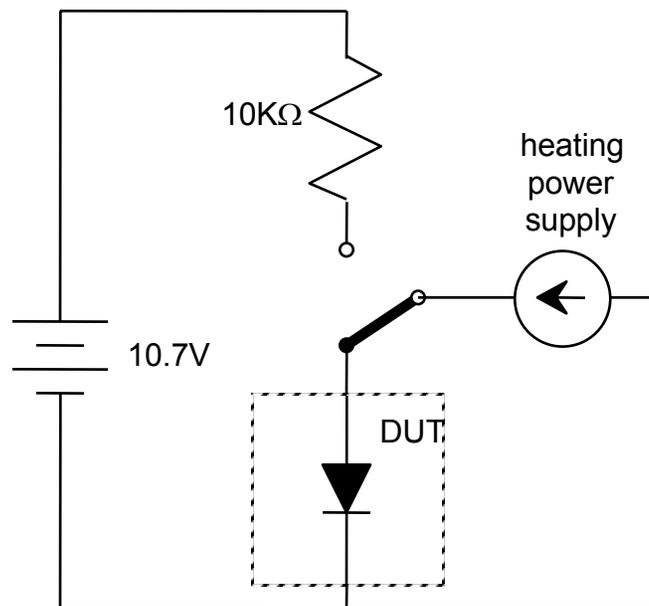
approximate const. current supply

OR



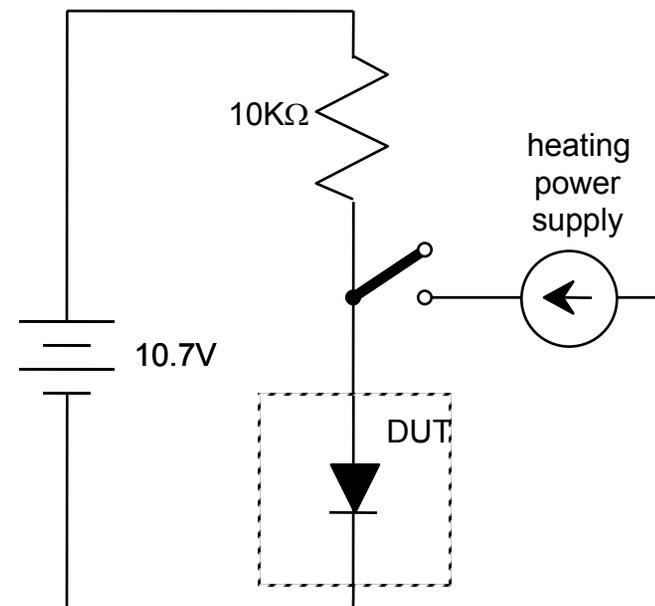
How to heat

sample current is off
while heating current on



OR

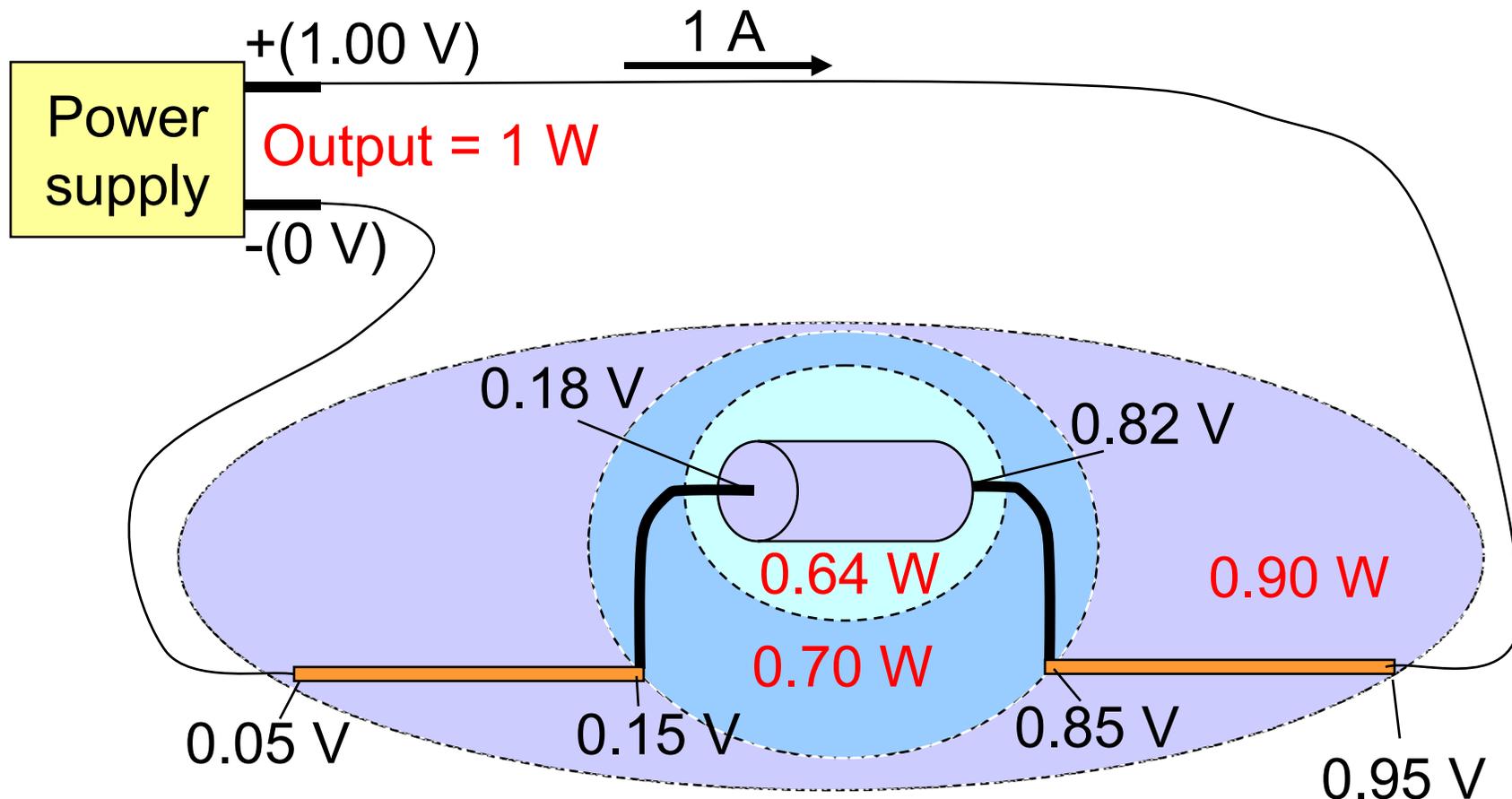
sample current
is always on



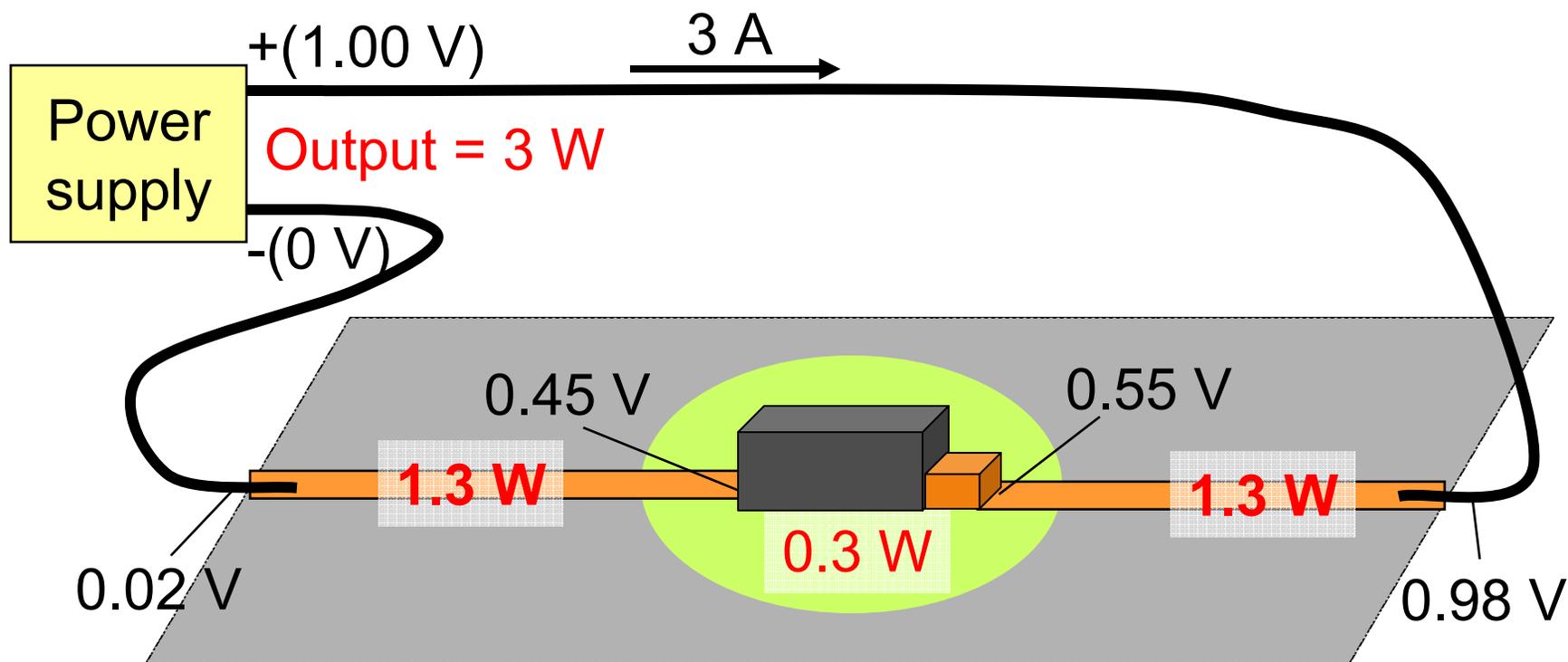
Superposition and TSP “self heating”

- Common warning:
 - Keep the TSP power low! “self heating is bad!”
- But is this really a problem?
 - If the “sample” power is always there, the “self heating” is the same during calibration as during test, so they cancel out
- You might unwittingly overheat the junction
- You might not be able to keep the “measurement” current on during the heating
 - But if this is a serious issue, reduce the effective “test” power by the amount of “measurement” power

The Importance of 4-wire measurements



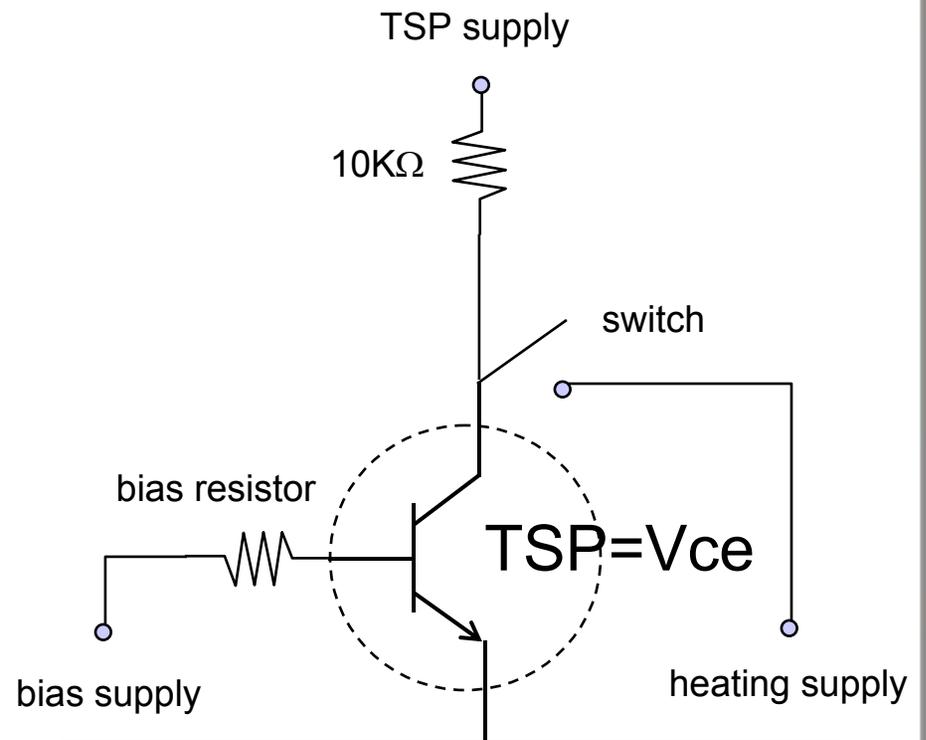
Which raises an interesting question:



Is this a fair characterization of a low- R_{ds-on} device?

Bipolar transistor

- TSP is V_{ce} at designated “constant” current
- Heating is through V_{ce}
- Choose a base current which permits adequate heating

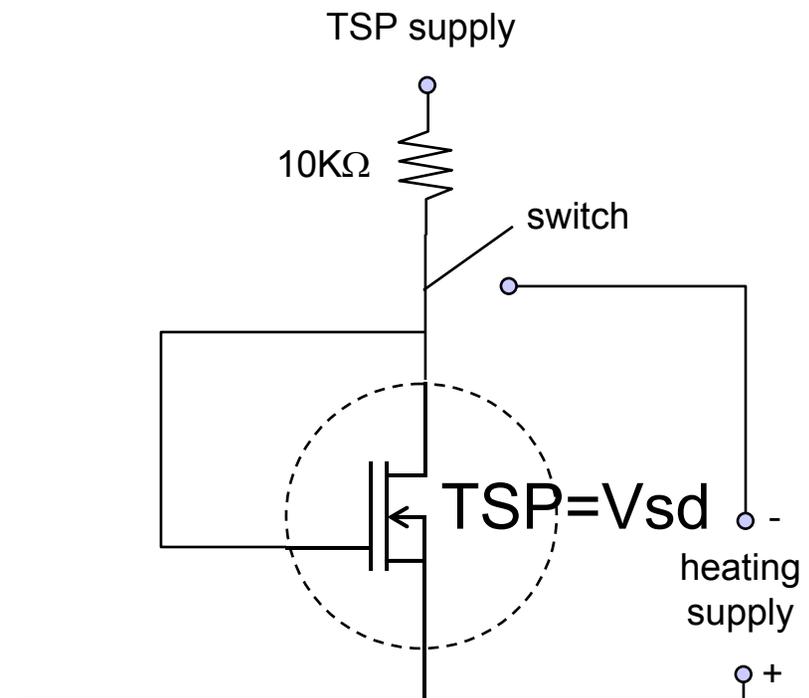


Schottky diode

- TSP is forward voltage at “low” current
- Voltages are typically very small (especially as temperature goes up)
- Highly non-linear, though maybe better as TSP current increases; because voltage is low, higher TSP current may be acceptable
- Heating current will be large

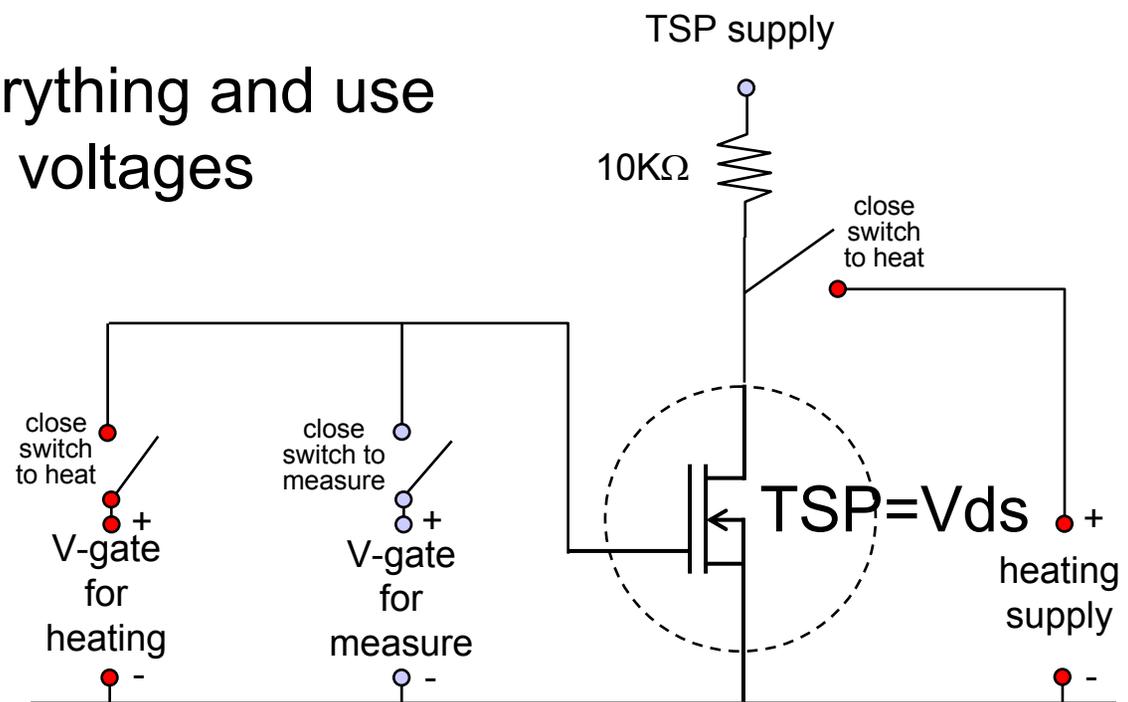
MOSFET / TMOS

- Typically, use reverse bias “back body diode” for both TSP and for heating
- May need to tie gate to source (or drain) for reliable TSP characteristic



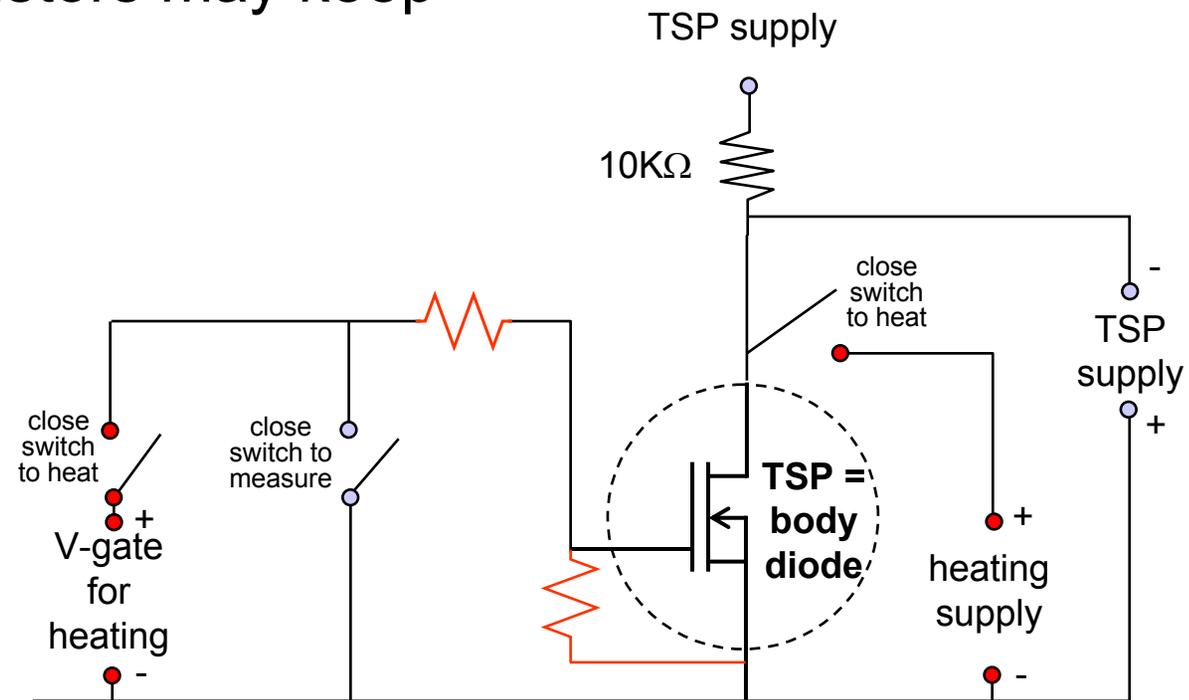
MOSFET / TMOS method 2

- If you have fast switches and stable supplies
- Forward bias everything and use two different gate voltages



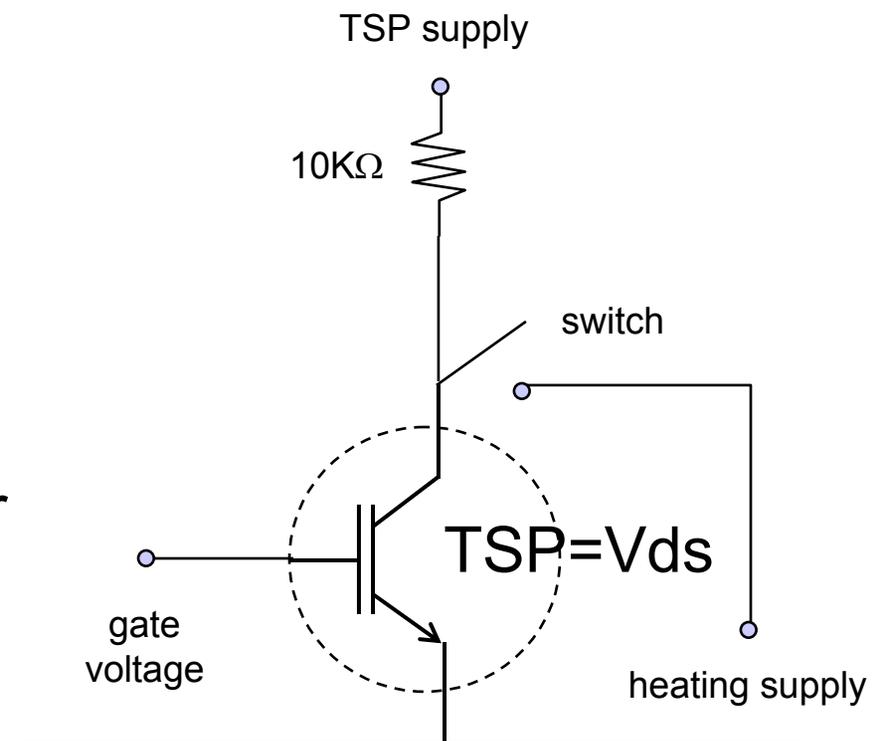
RF MOS

- They exist to amplify high frequencies (i.e. noise)!
- Feedback resistors may keep them in DC



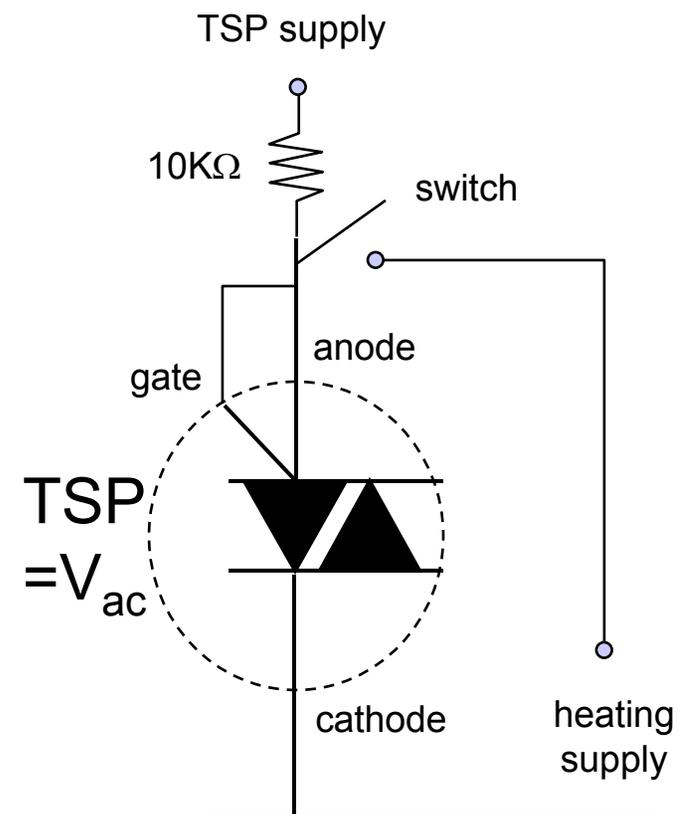
IGBT

- Drain-source channel used for both TSP and heating
- Find a gate voltage which “turns on” the drain-source channel enough for heating purposes
- Use same gate voltage, but typically low TSP current for temperature measurement



Thyristor

- Anode--to-cathode voltage path used both for TSP and for heating
- typical TSP current probably lower than “holding” current, so gate must be turned on for TSP readings; try tying it to the anode (even so, we used 20mA to test SCR2146)
- Hopefully, with anode tied to gate, enough power can be dissipated to heat device without exceeding gate voltage limit



Logic and analog

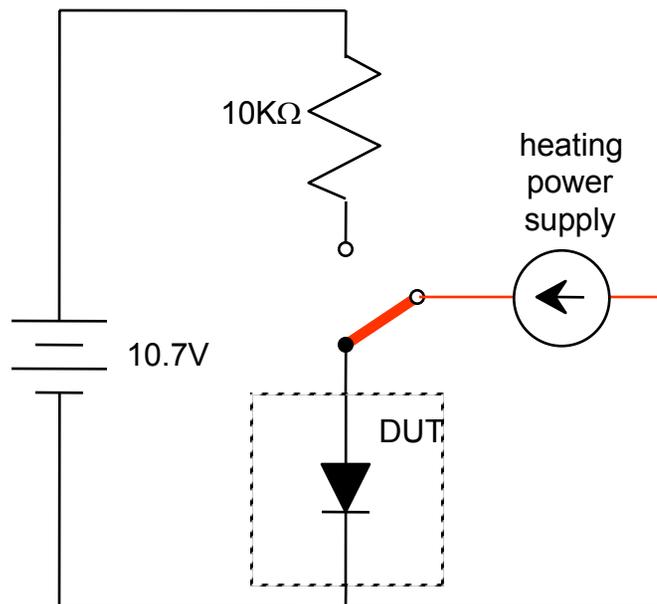
- Find any TSP you can
 - ESD diodes on inputs or outputs
 - Body diodes somewhere
- Heat wherever you can
 - High voltage limits on V_{cc} , V_{ee} , whatever
 - Body diodes or output drivers
 - Live loads on outputs
 - (be very careful how you measure power!)



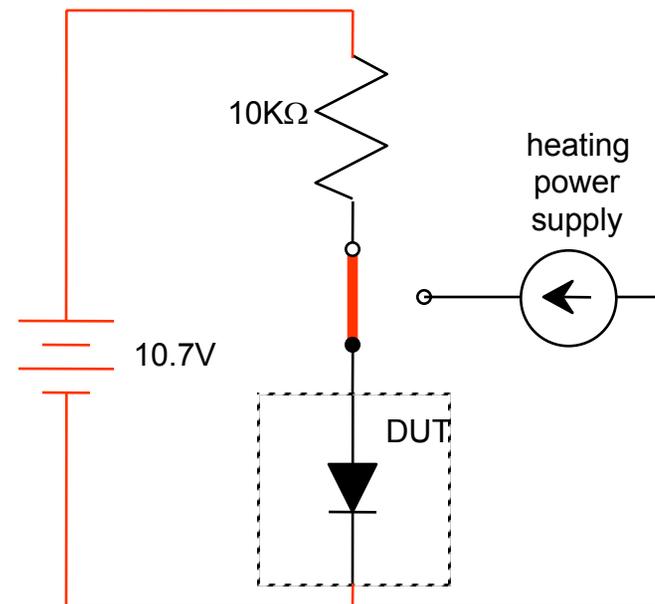
Heating curve method vs. cooling curve method

Quick review: Basic T_j measurement

first we heat



then we measure



Question

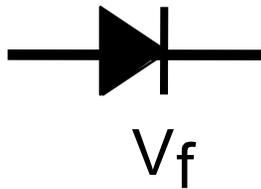
- What happens when you switch from “heat” to “measure”?

Answer: stuff changes

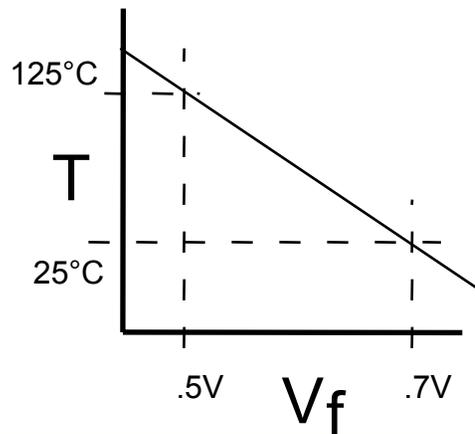
- More specifically, while the electrical signal is stabilizing, the junction starts to cool down



Basic “Heating Curve” Transient Method

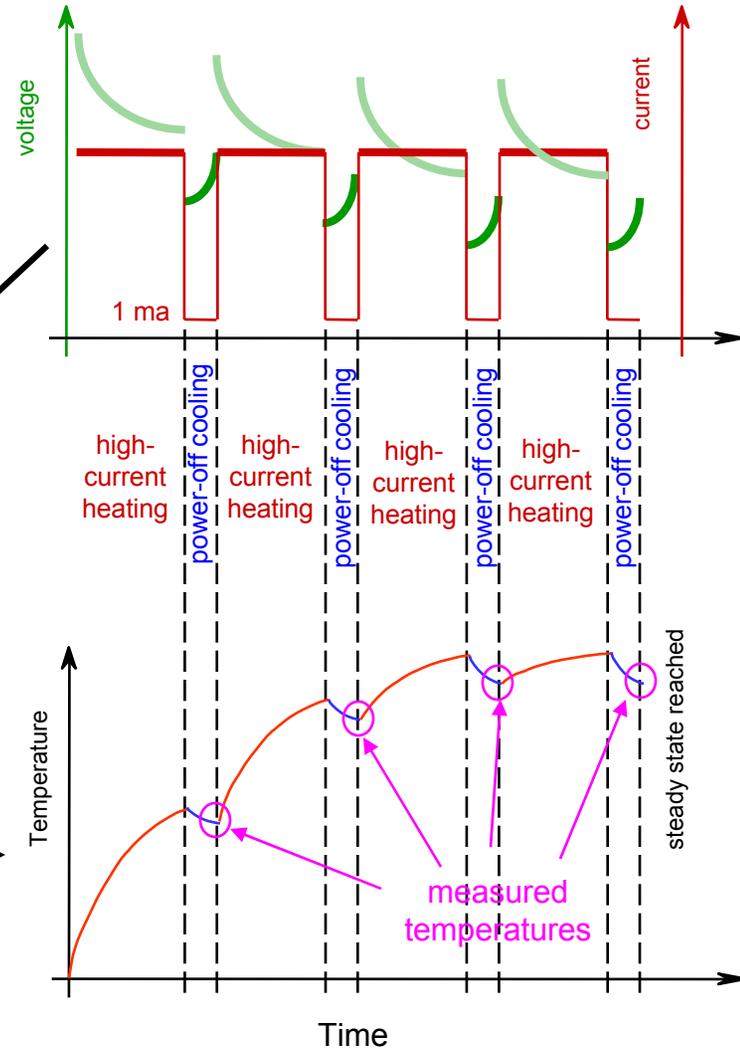


calibrate forward voltage
@ 1mA sense current



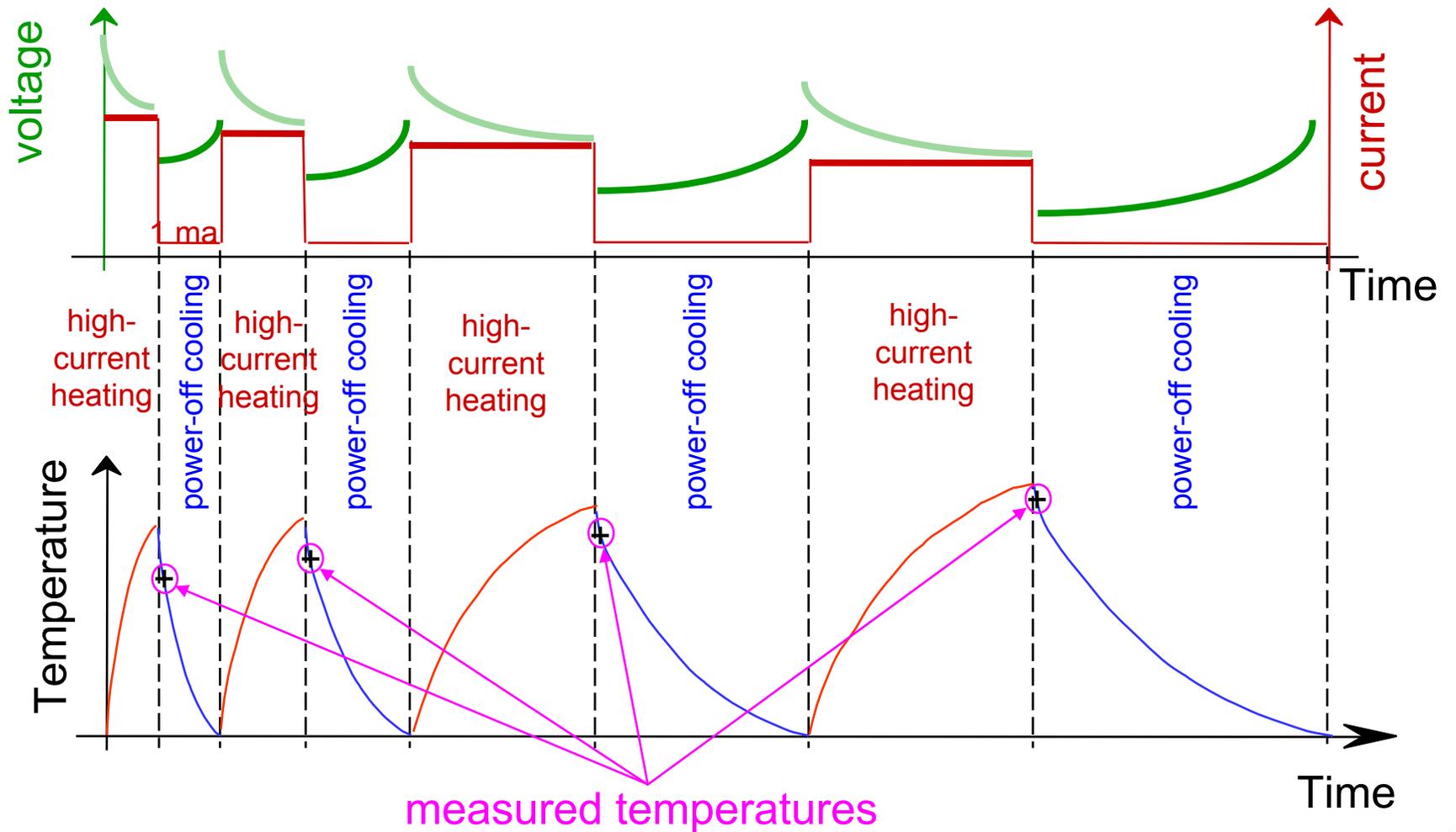
convert cooling
volts to
temperature

measurements



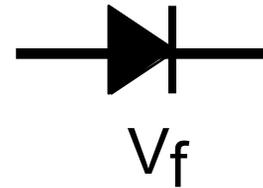
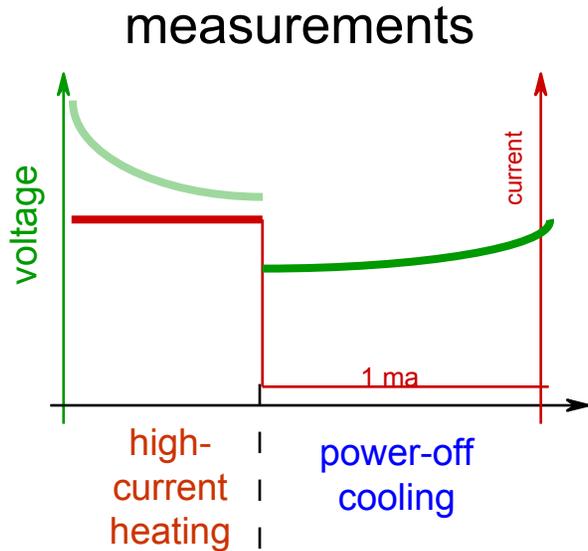


Heating curve method #2

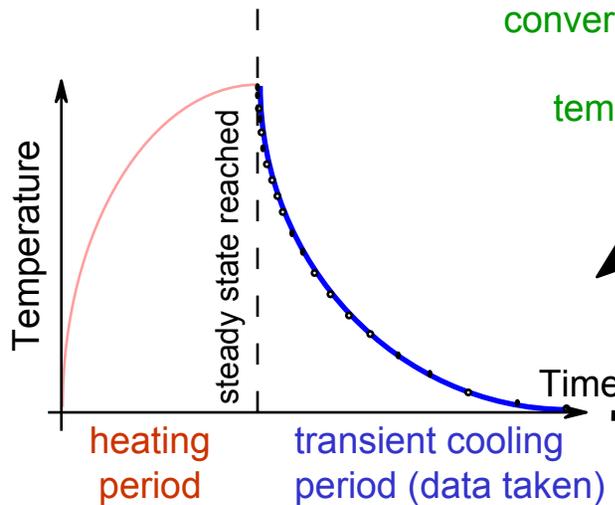




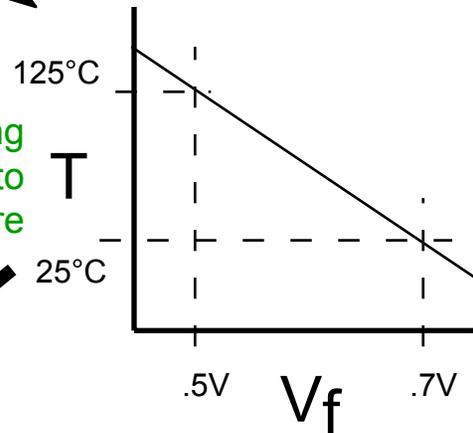
Basic "Cooling Curve" Transient Method



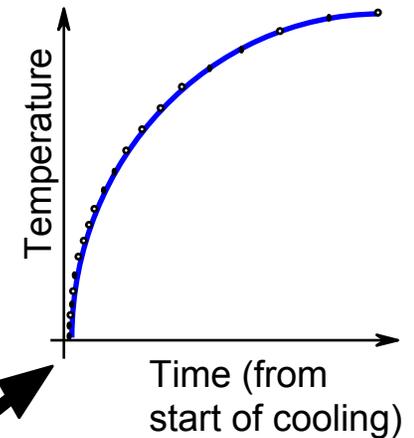
calibrate forward voltage
@ 1mA sense current



convert cooling
volts to
temperature



subtract cooling curve from
peak temperature to obtain
"heating" curve equivalent

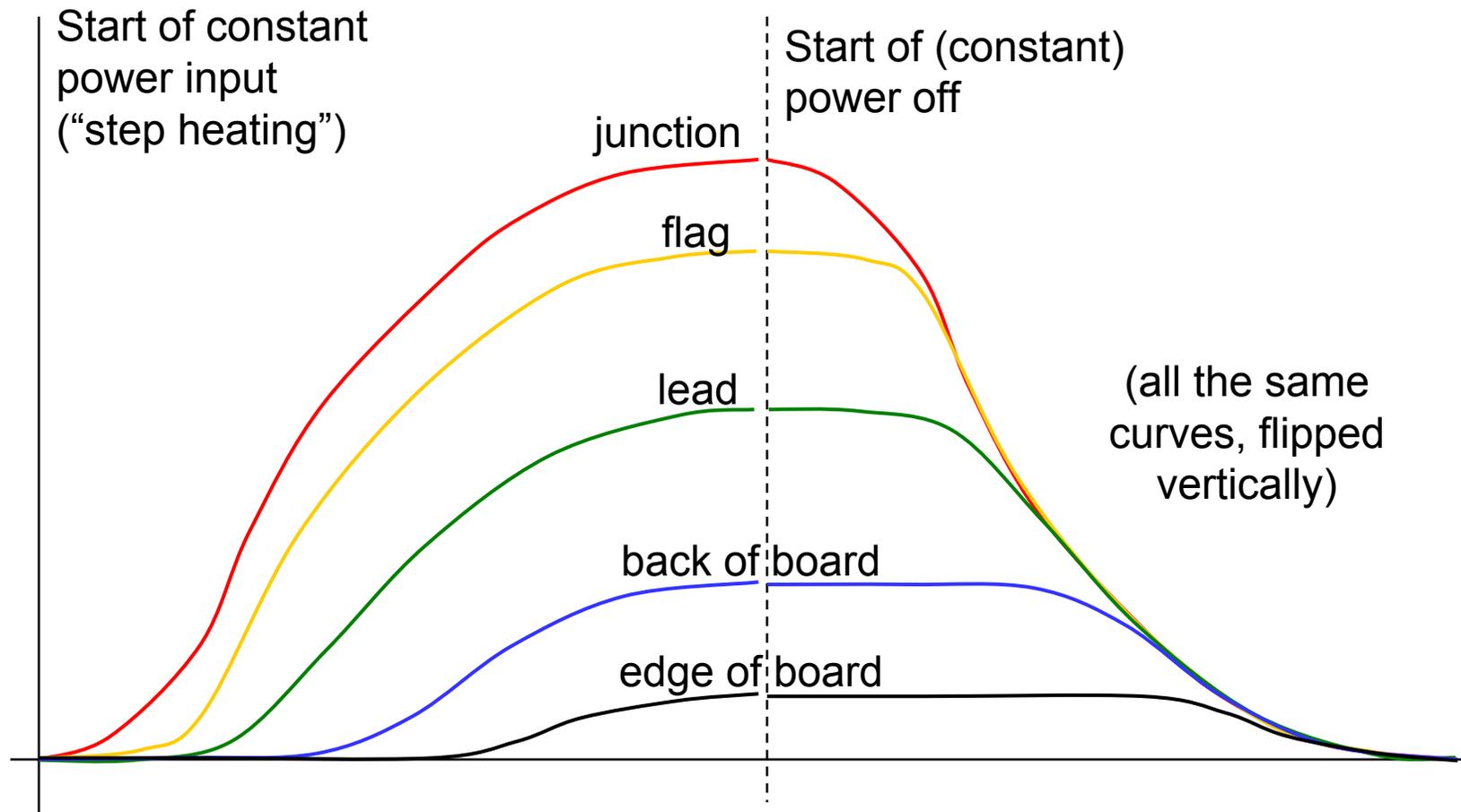


Whoa!

... that last step, there ...

- Heating vs. cooling
 - Physics is symmetric, **as long as the material and system properties are independent of temperature**

Heating vs. cooling symmetry



(cooling)

- For a theoretically valid cooling curve, you must begin at true thermal equilibrium (not uniform temperature, but steady state)
- So whatever your θ_{JA} , max power is limited to:

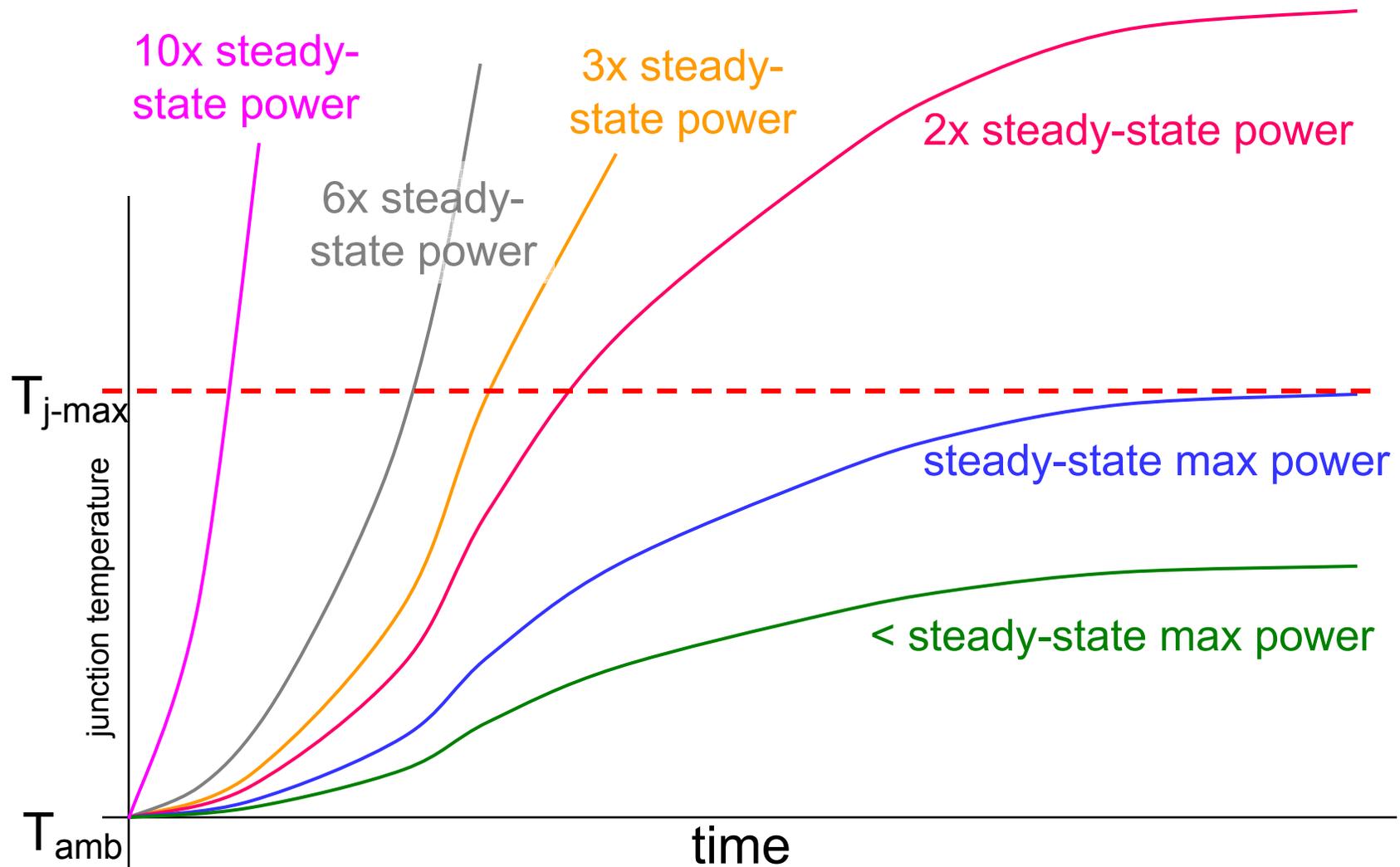
$$power = \frac{T_{j\max} - T_{\text{ambient}}}{\theta_{JA}}$$

(cooling)

By the way ...

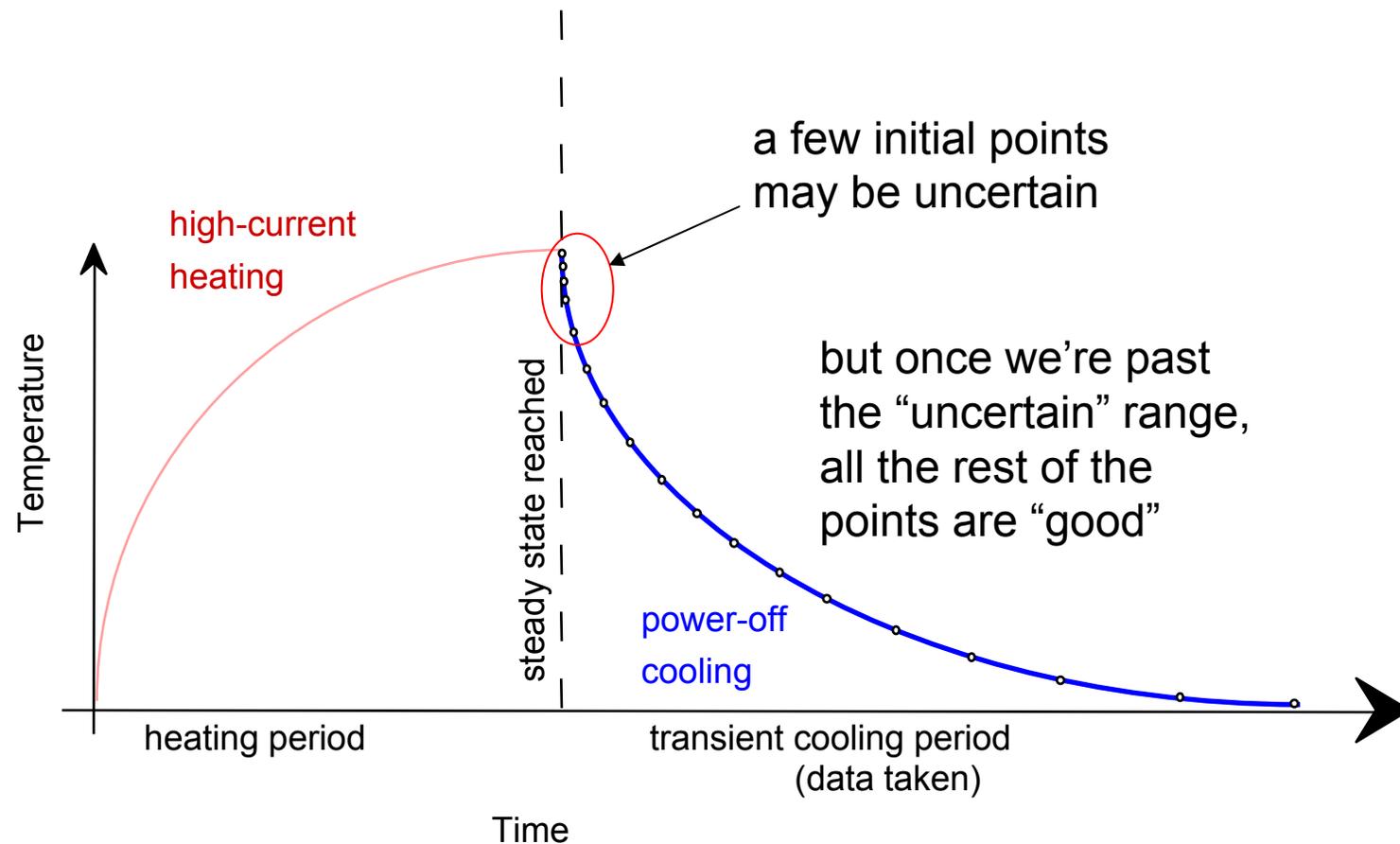
- Since you must have the device at steady state in order to make a full transient cooling-curve measurement, steady-state θ_{JA} is a freebie.
(given that you account for the slight cooling which took place before your first good measurement occurred)

Effect of power on heating curve



(cooling)

Some initial uncertainty



Heating vs. cooling tradeoffs

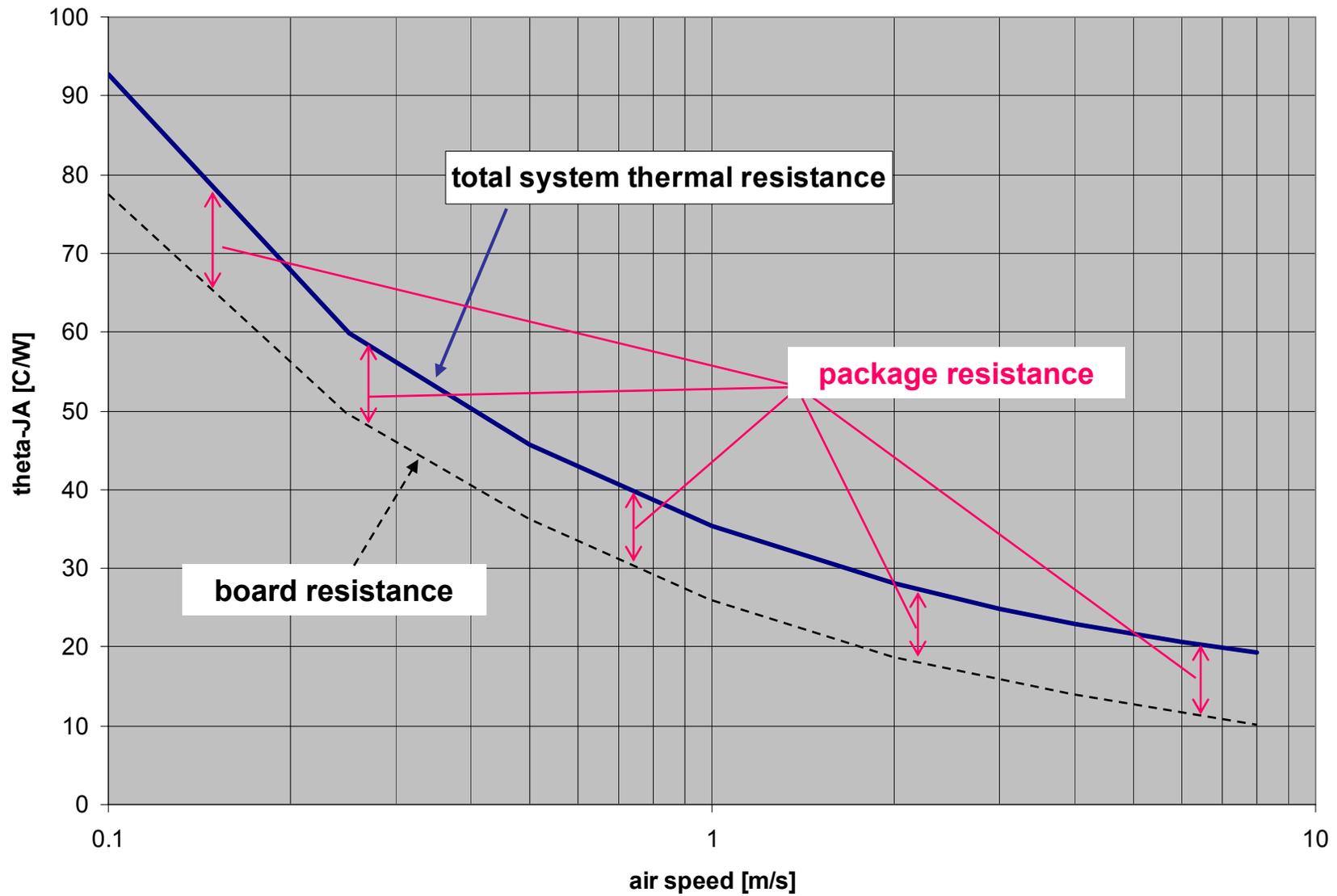
	HEATING	COOLING
starting temperature	ambient	?
heating power	limited by tester	limited to steady-state
temperature of fastest data	closer to ambient	closer to T_{j-max}
error control	all points similar error	error limited to first few points

Test Conditions

- Still air, moving air
- Various mounting configurations
 - Min-pad board
 - 1” heat spreader board
- Coldplate testing
 - Single, dual, “ring”

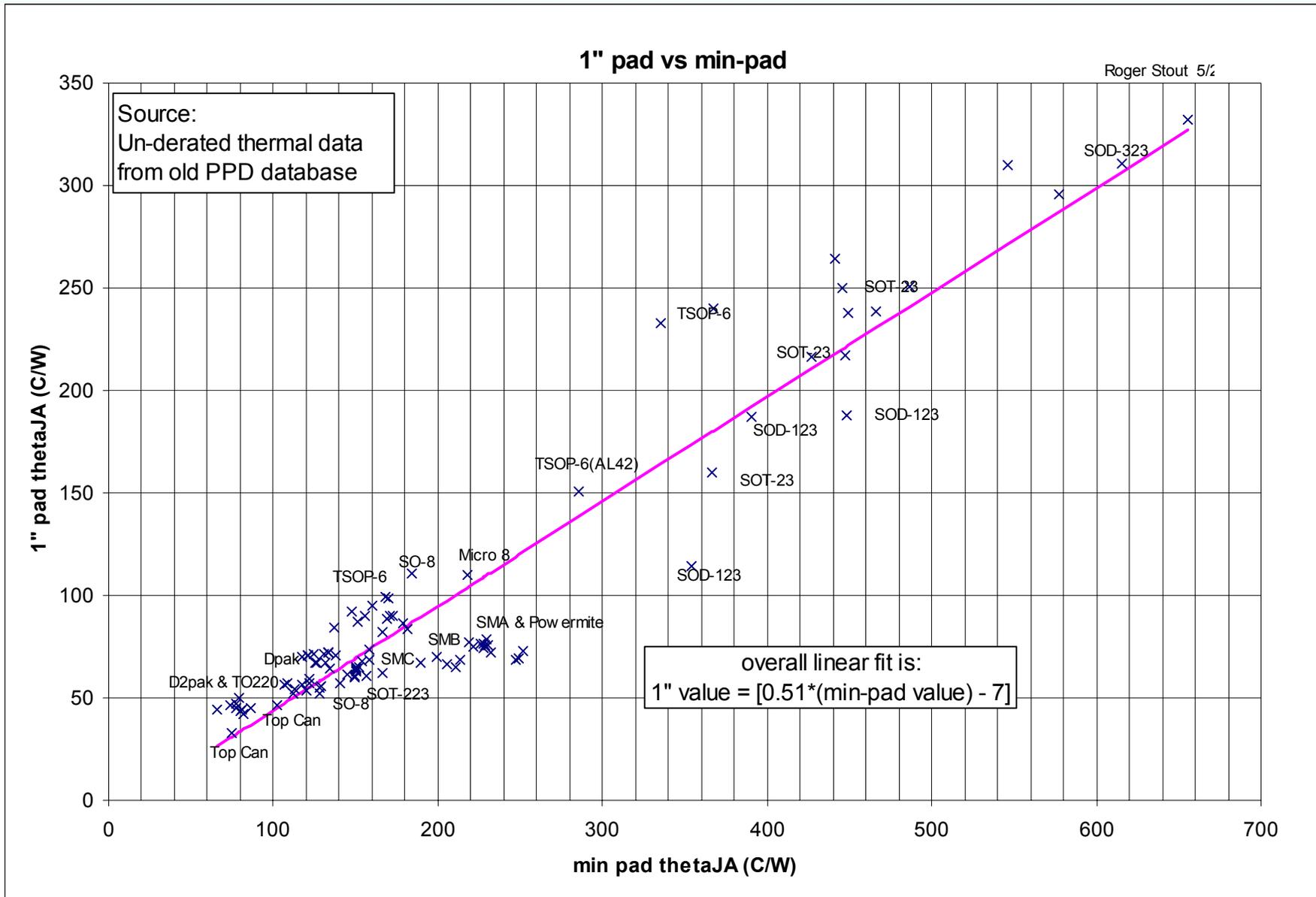
Still air vs. moving air

- Varying the air speed is mainly varying the heat loss from the test board surface area, not from the package itself
- You just keep re-measuring your board's characteristics



Different boards

- min-pad board
- 1" heat spreader board
- you're mainly characterizing how copper area affects *every* package and board, not how a *particular* package depends on copper area

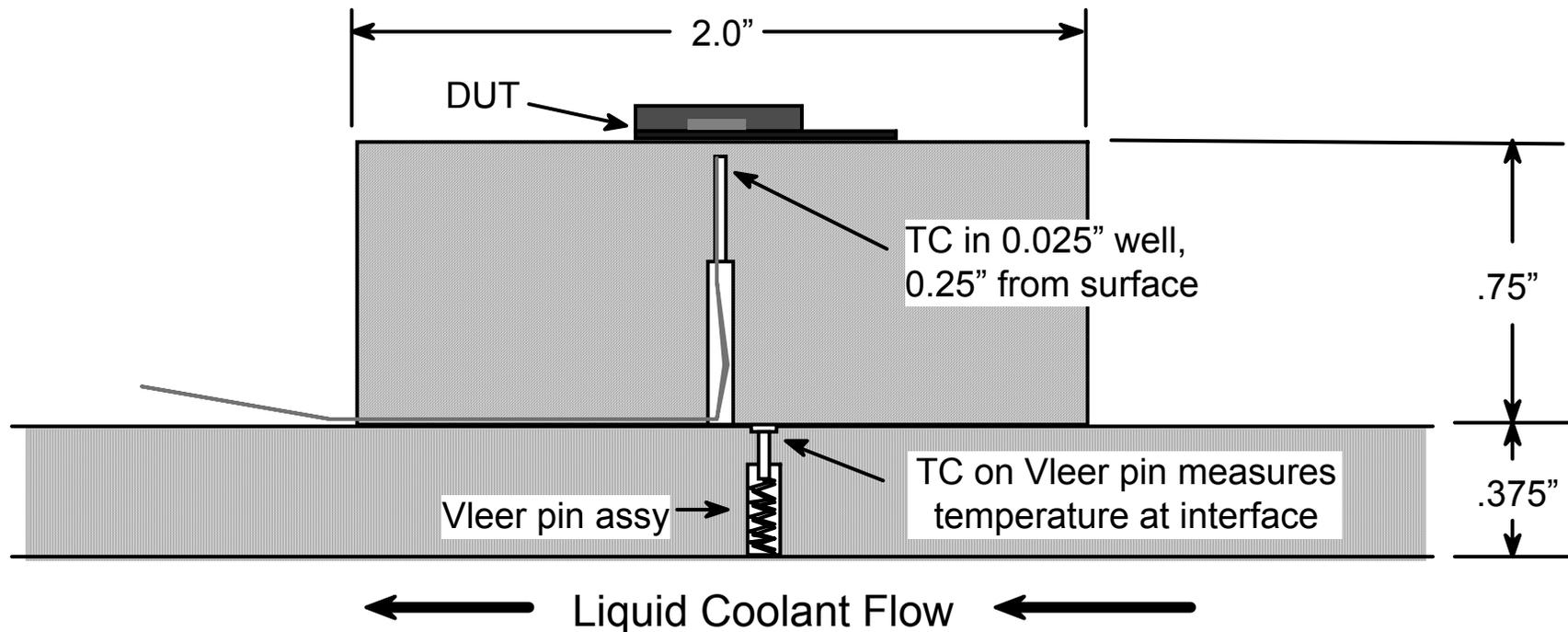


Standard coldplate testing

- “infinite” heatsink (that really isn’t) for measuring theta-JC on high-power devices
- If both power and coldplate temperature are independently controlled, “two parameter” compact models may be created

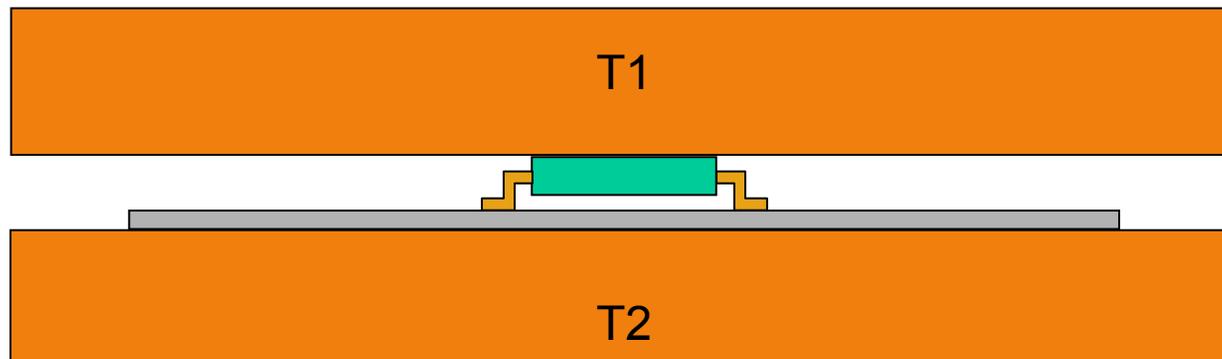
Standard coldplate testing

- Detailed design and placement of “case” TC can have significant effect on measured value



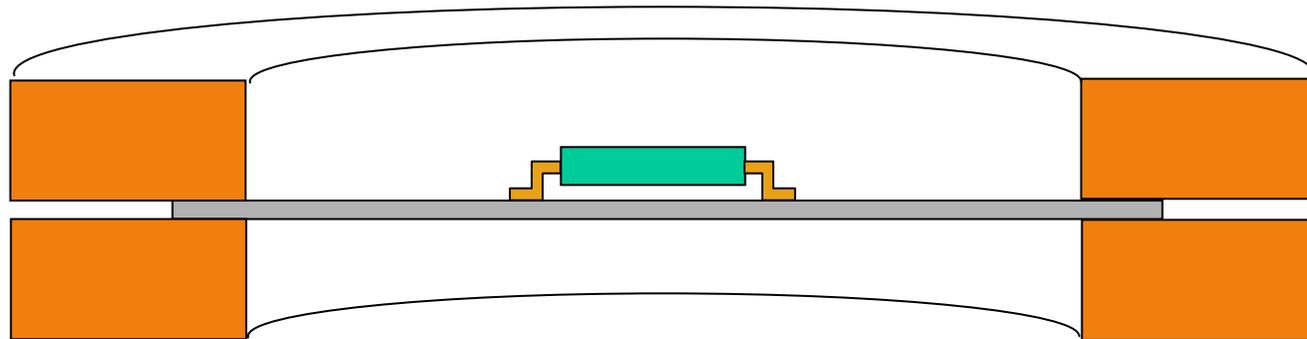
“Dual” coldplate testing

- Alternative method for “two-parameter” characterization methods where two independent “isothermal” boundary conditions are desired

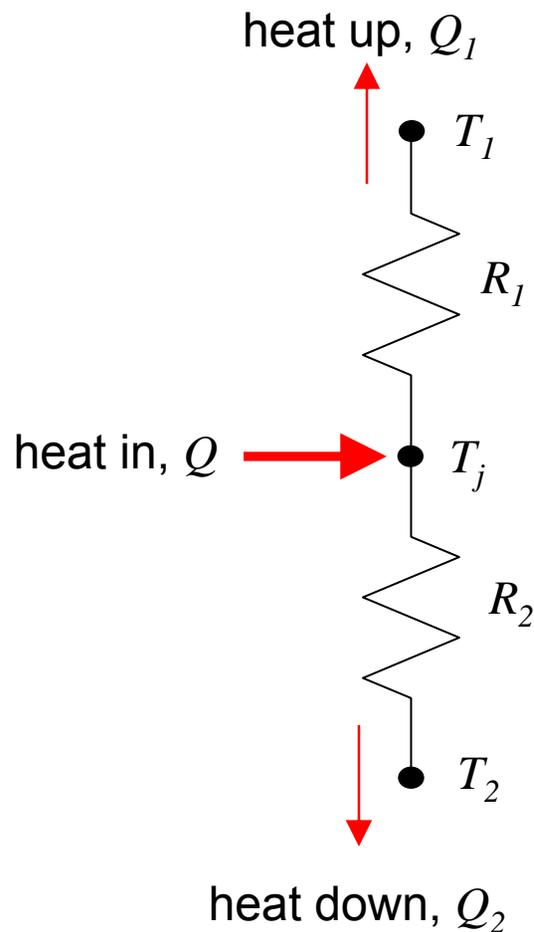


“Ring” coldplate

- For making somewhat higher-power board-mounted measurements; “ring” coldplate is clamped around outer edge of test board to constrain board temperature



2-parameter data reduction



$$Q = Q_1 + Q_2$$

$$Q = \frac{1}{R_1}(T_j - T_1) + \frac{1}{R_2}(T_j - T_2)$$

This has the form of a two-variable linear equation:

$$y = m_1x_1 + m_2x_2 + b$$

where:

$$m_1 = \frac{1}{R_1} \quad x_1 = (T_j - T_1) \quad b \equiv 0$$

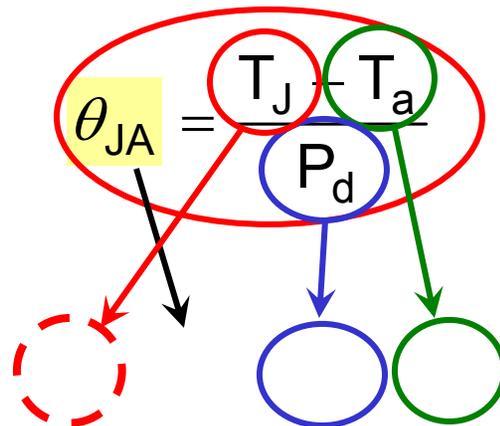
$$m_2 = \frac{1}{R_2} \quad x_2 = (T_j - T_2)$$

What's wrong with theta-JA?

THERMAL RATINGS

Parameter	Test Conditions Typical Value		Units
	Min-Pad Board (Note 1)	1.0 in Pad Board (Note 2)	
Junction-to-Tab (ψ_{JL2}, Ψ_{JL2}) (Note 3)	48	43	°C/W
Junction-to-Ambient ($R_{\theta JA}, \theta_{JA}$)	183	120	°C/W

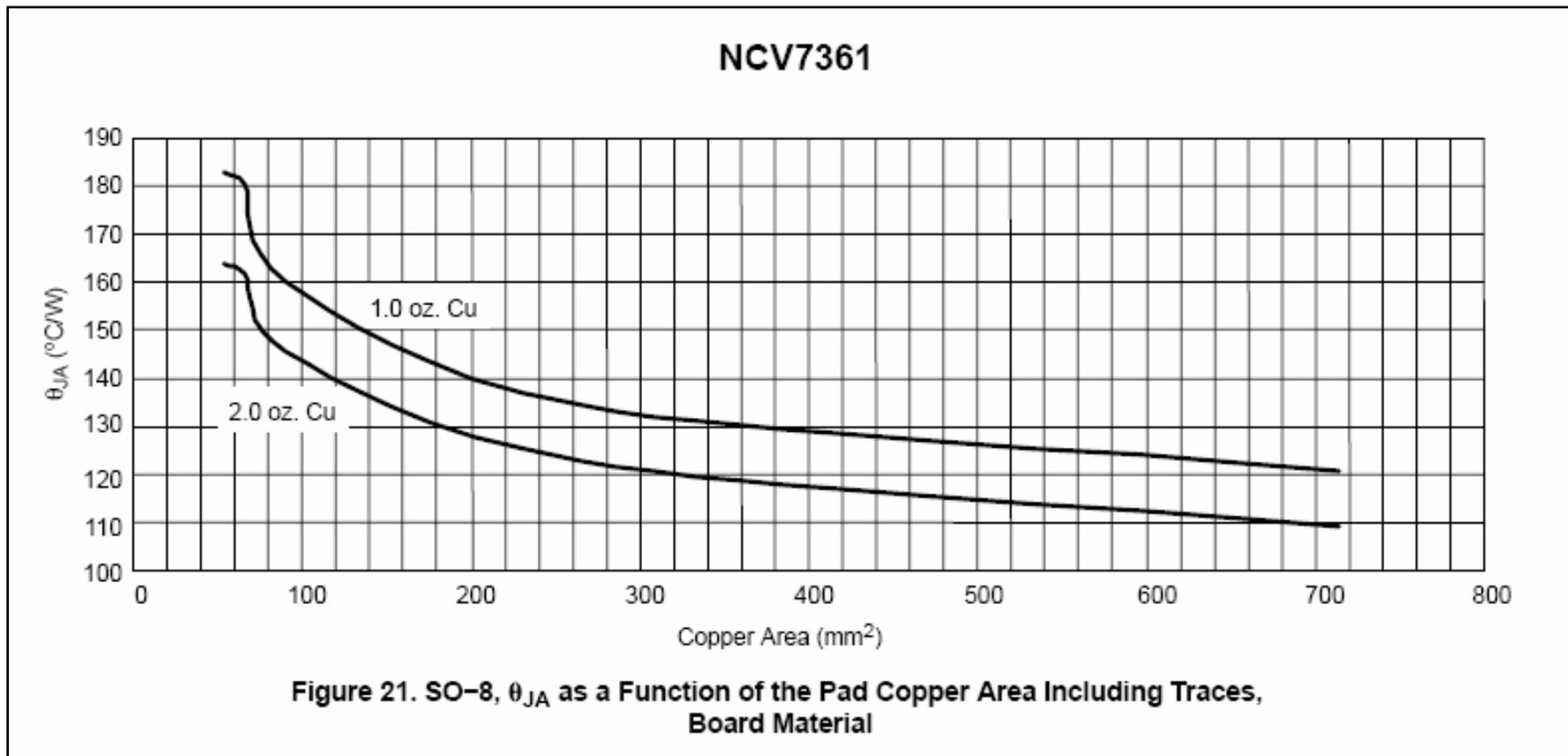
- 1 oz copper, 54 mm² copper area, 0.062" thick FR4.
- 1 oz copper, 714 mm² copper area, 0.062" thick FR4.
- ψ_{JL2} temperature was made at foot of lead #2.



$$\Psi_{Jtab} = \frac{T_J - T_{tab}}{P_d}$$

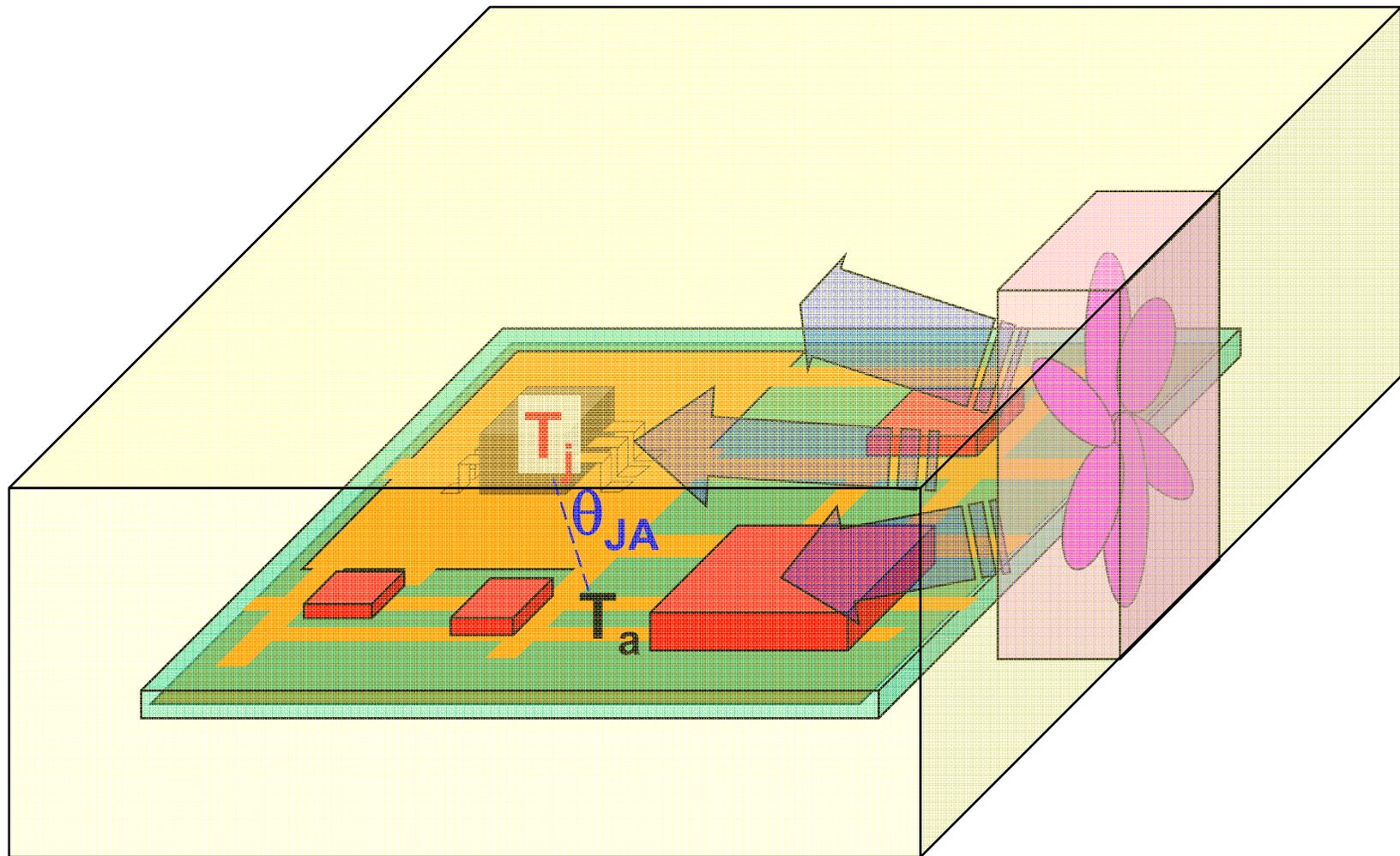
$$T_J = \Psi_{Jtab} \cdot P_d + T_{tab}$$

Theta-JA vs copper area





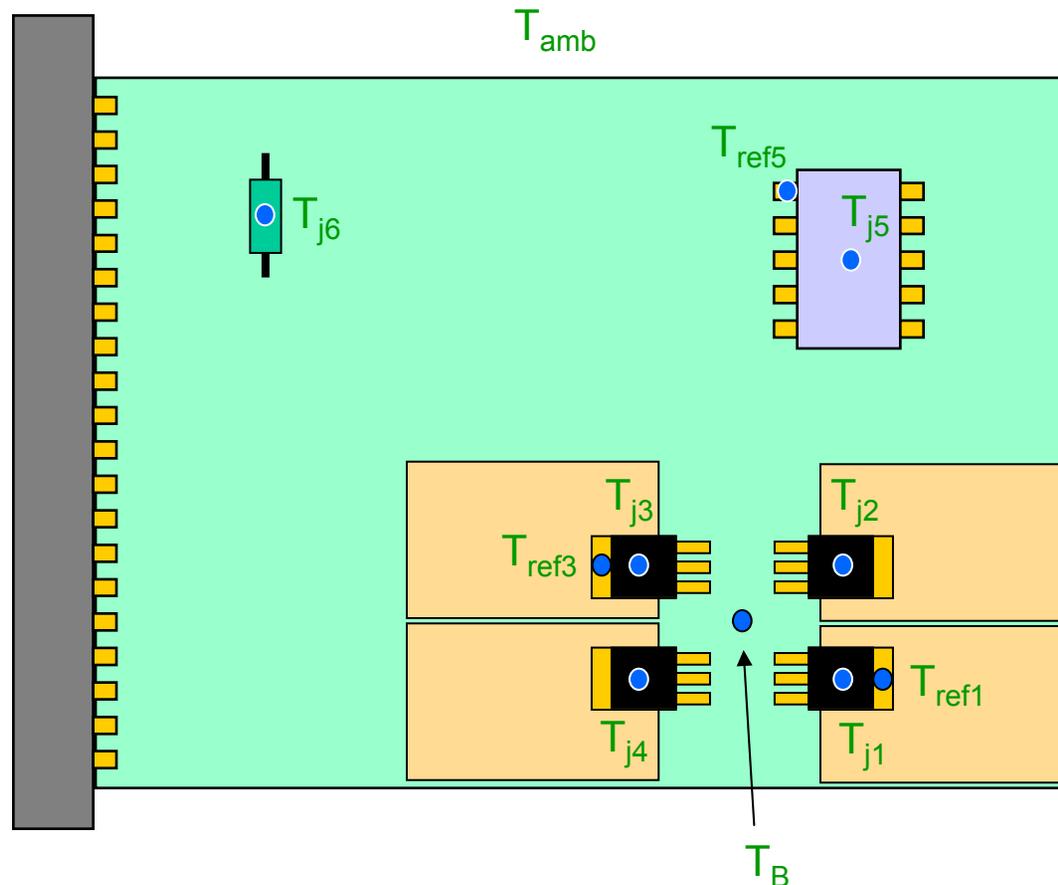
Linear superposition



Facts and fallacies redux

- Basic idea:
 - “thermal resistance” is an intrinsic property of a package
- Flaws in idea:
 - there is no isothermal “surface”, so you can’t define a “case” temperature
 - Plastic body (especially) has big gradients
 - different leads are at different temperatures
 - multiple, parallel thermal paths out of package
 - other heat sources change everything

Our case-study will be this 6-component thermal system



Linear superposition – what is it?

- The total response of a point within the system, to excitations at all points of the system, is the sum of the individual responses to each excitation taken independently.

$$\Delta T_{\text{composite}} = \Delta T_{\text{source 1}} + \Delta T_{\text{source 2}} + \cdots + \Delta T_{\text{source n}}$$

Linear superposition – when does it apply?

- The system must be “linear” – in brief, all individual responses must be proportional to all individual excitations.

$$\begin{aligned} \Delta T_{\text{net A}} &= \Delta T_{A \leftarrow B} + \Delta T_{A \leftarrow C} + \Delta T_{A \leftarrow D} \\ &= \underbrace{2 \cdot q_B}_{\text{blue}} + \underbrace{3 \cdot q_C}_{\text{blue}} + \underbrace{1.2 \cdot q_D}_{\text{blue}} \end{aligned}$$

**Linear superposition doesn't
apply if the system isn't *linear*.**

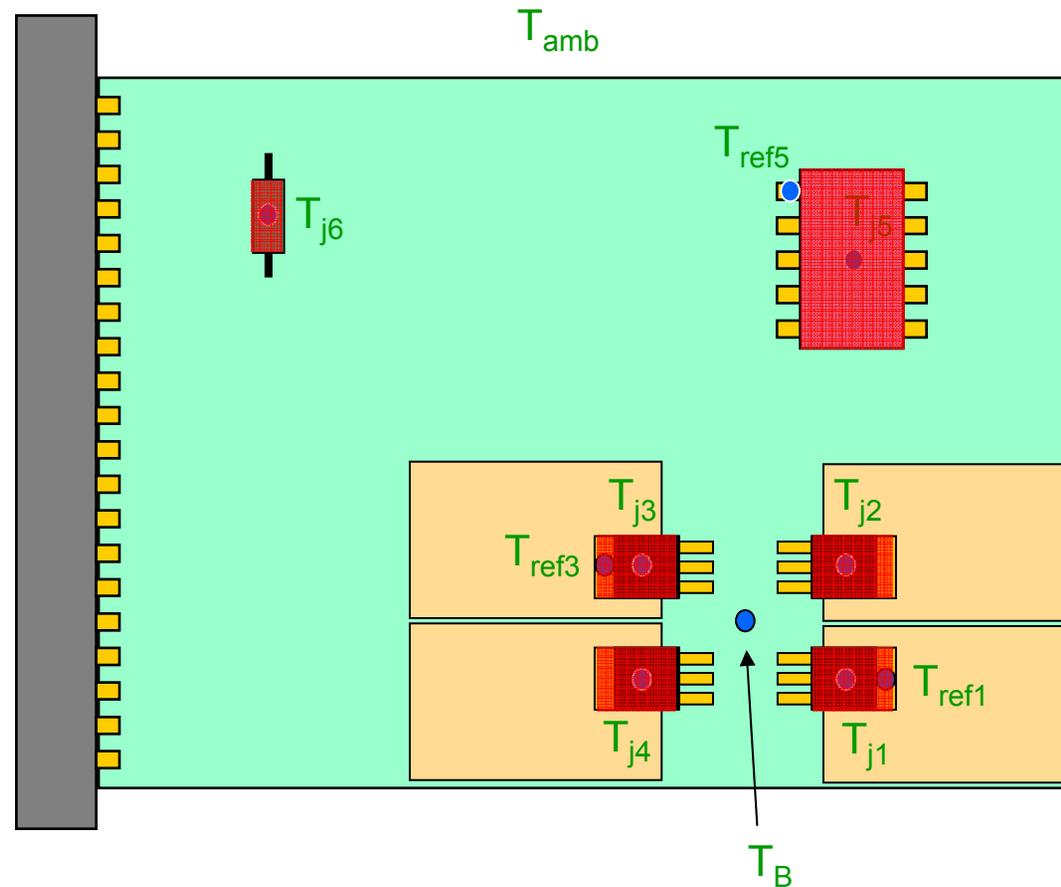
$$\Delta T = \underbrace{a(T, q_1)} \cdot q_1 + \underbrace{b(T, q_2)} \cdot q_2 + \dots$$

$$\Delta T = a \cdot q_1^{n1} + b \cdot q_2^{n2} + \dots$$

Linear superposition – when would you use it?

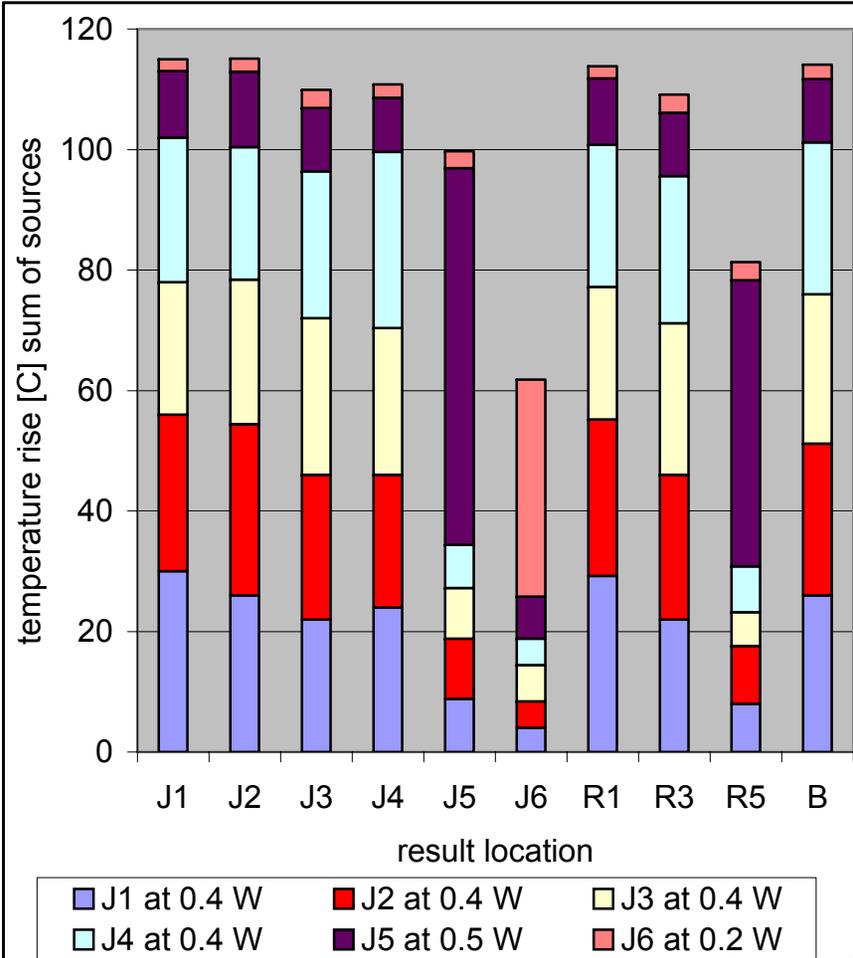
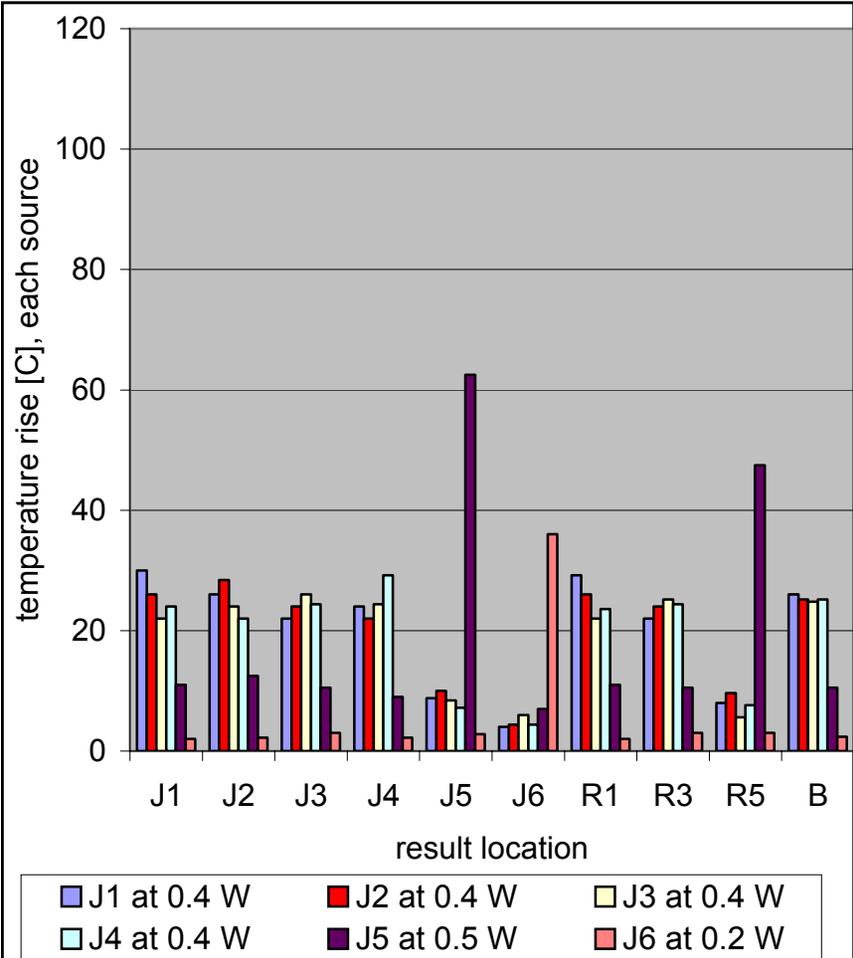
When you have multiple heat sources
(that is, all the time!)

Linear superposition – how do you use it?

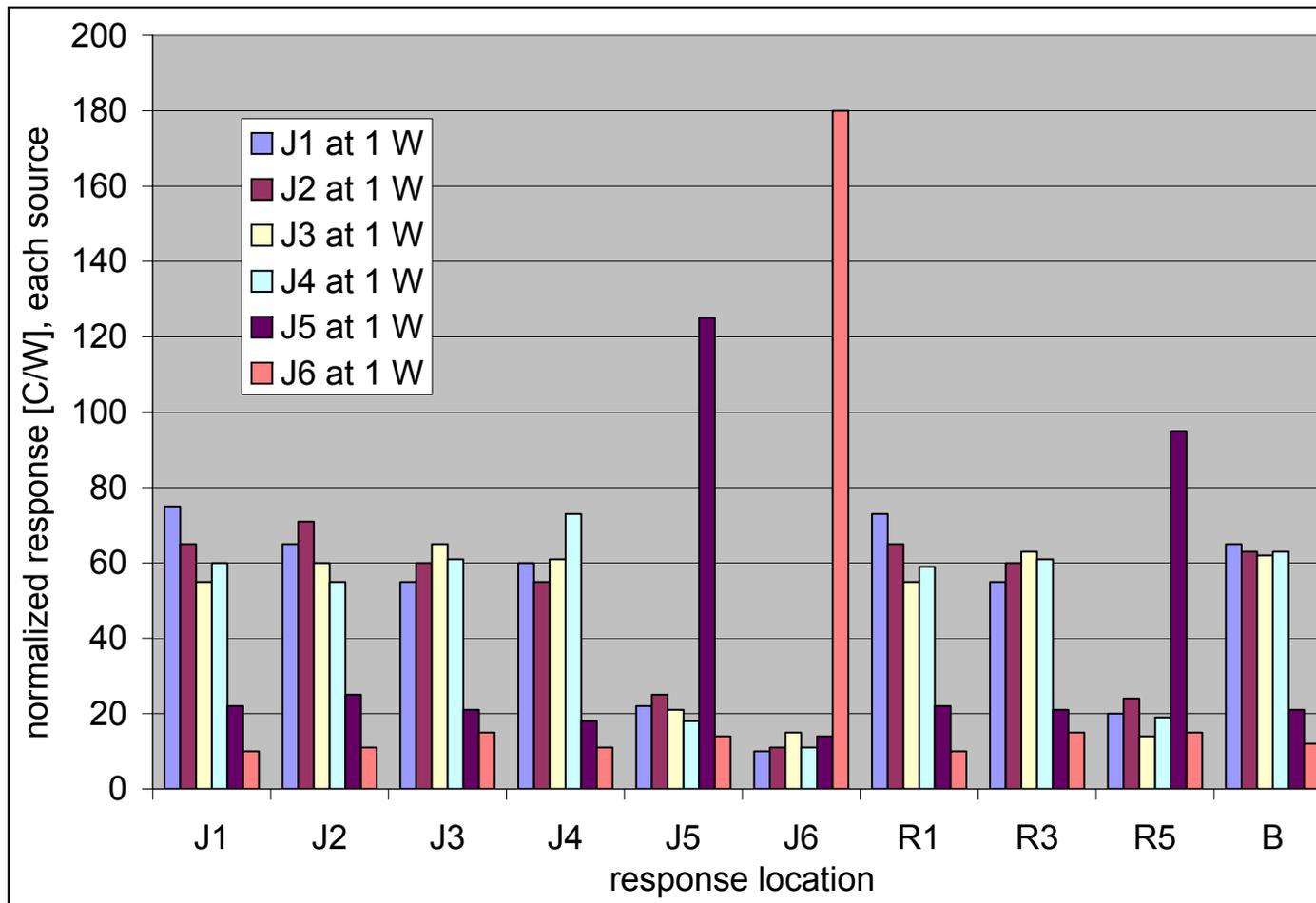




Temperature direct contributions and totals



Normalized responses at each location due to each source





junction temperature vector

theta matrix assembled from simplified subsystems

power input vector

$$\begin{Bmatrix} T_{j1} \\ \mathbf{T}_J \\ T_{jn} \end{Bmatrix} = \begin{bmatrix} \theta_{J1A} & \Psi_{12} & \dots & \Psi_{1n} \\ \Psi_{12} & \mathbf{\theta}_{JA} & & \Psi_{2n} \\ \vdots & & & \vdots \\ \Psi_{1n} & \Psi_{2n} & \dots & \theta_{JnA} \end{bmatrix} \begin{Bmatrix} q_1 \\ \mathbf{q} \\ q_n \end{Bmatrix} + T_a$$

self-heating terms

board interactions



junction
temperature
vector

theta matrix assembled
from simplified subsystems

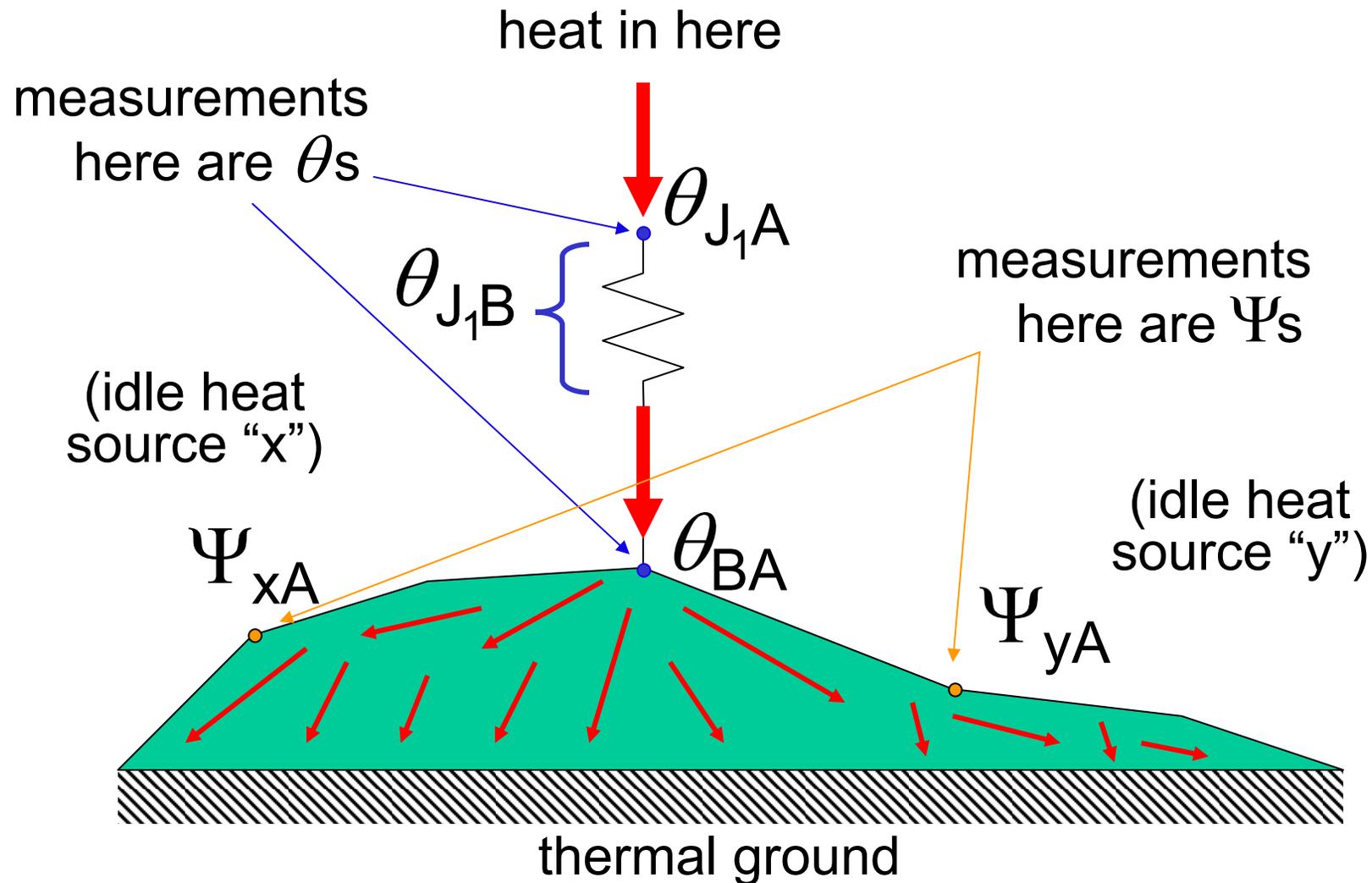
power
input
vector

$$\begin{Bmatrix} T_{j1} \\ T_{j2} \\ \vdots \\ T_{jn} \end{Bmatrix} = \begin{bmatrix} \theta_{JB1} + \theta_{BA1} & \Psi_{12} & \dots & \Psi_{1n} \\ \Psi_{12} & \theta_{JB2} + \theta_{BA2} & & \Psi_{2n} \\ \vdots & & \ddots & \vdots \\ \Psi_{1n} & \Psi_{2n} & \dots & \theta_{JBn} + \theta_{BAn} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix} + T_a$$

device
resistance

board
resistance

visualizing theta and psi



theta matrix doesn't have to be square

junction
temperature
vector

*one column for
each heat source*

power input
vector

$$\begin{pmatrix} \theta_{JA1} & \Psi_{21} & \Psi_{31} \\ \Psi_{12} & \theta_{JA2} & \Psi_{32} \\ \Psi_{1x} & \Psi_{2x} & \Psi_{3x} \\ \Psi_{1L1} & \Psi_{2L1} & \Psi_{3L1} \\ \Psi_{1B} & \Psi_{2B} & \Psi_{3B} \end{pmatrix}$$

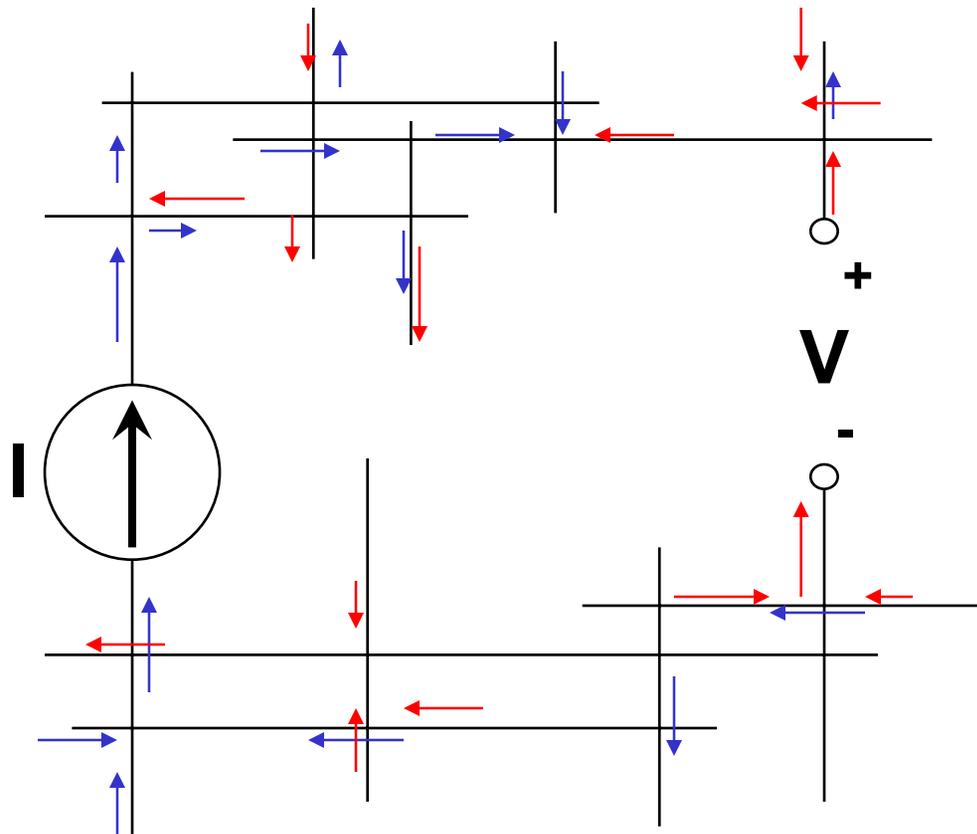
*one row
for each
heat
source*

*one row for each temperature
location of interest*

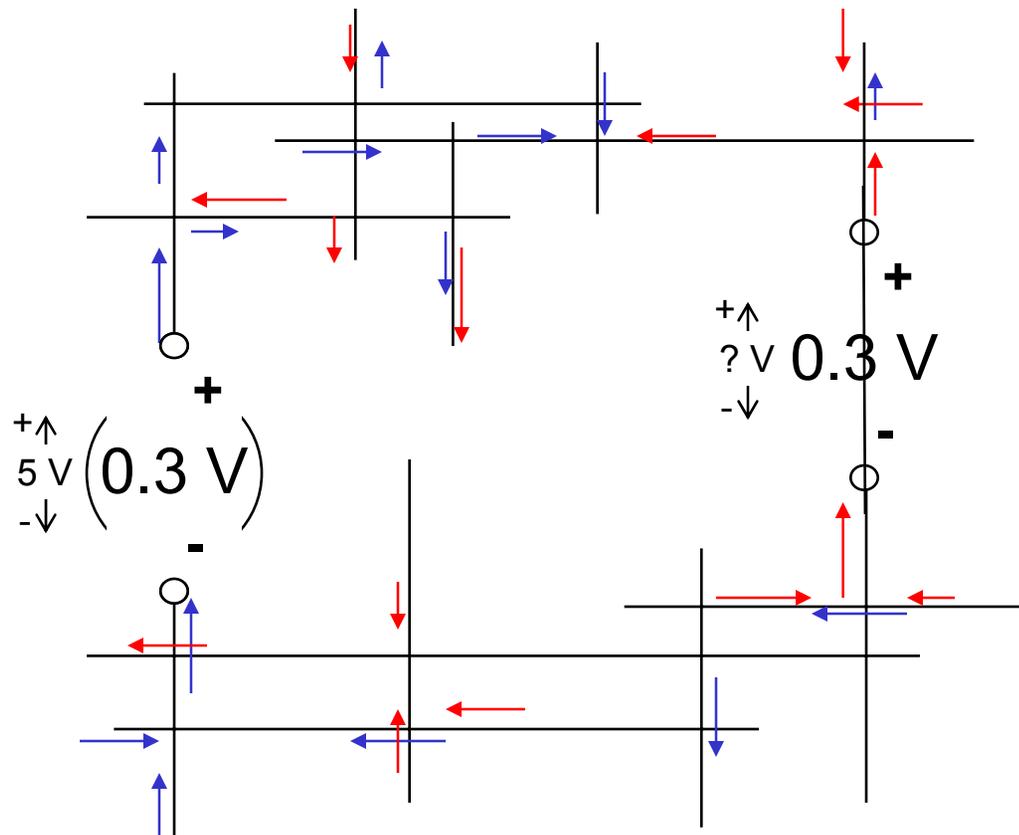
The reciprocity theorem

- What is it?
- When does it *not* apply?

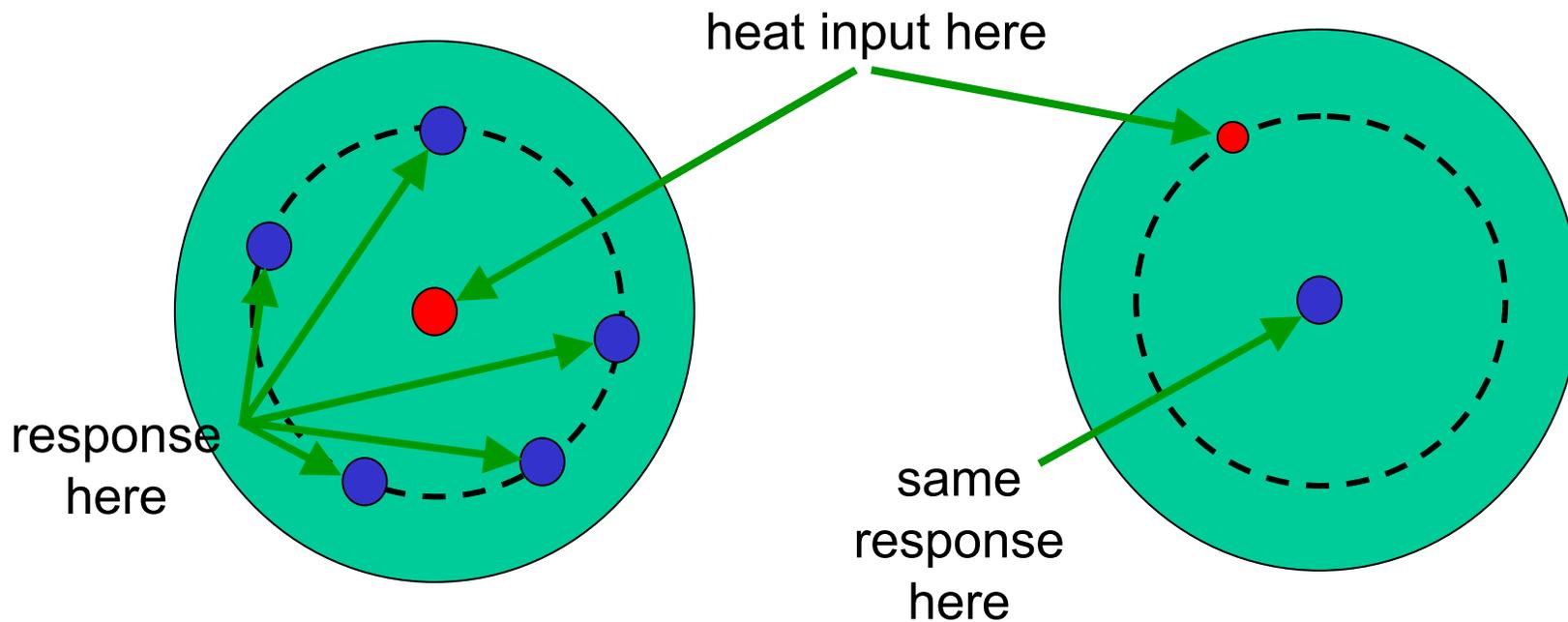
Electrical reciprocity



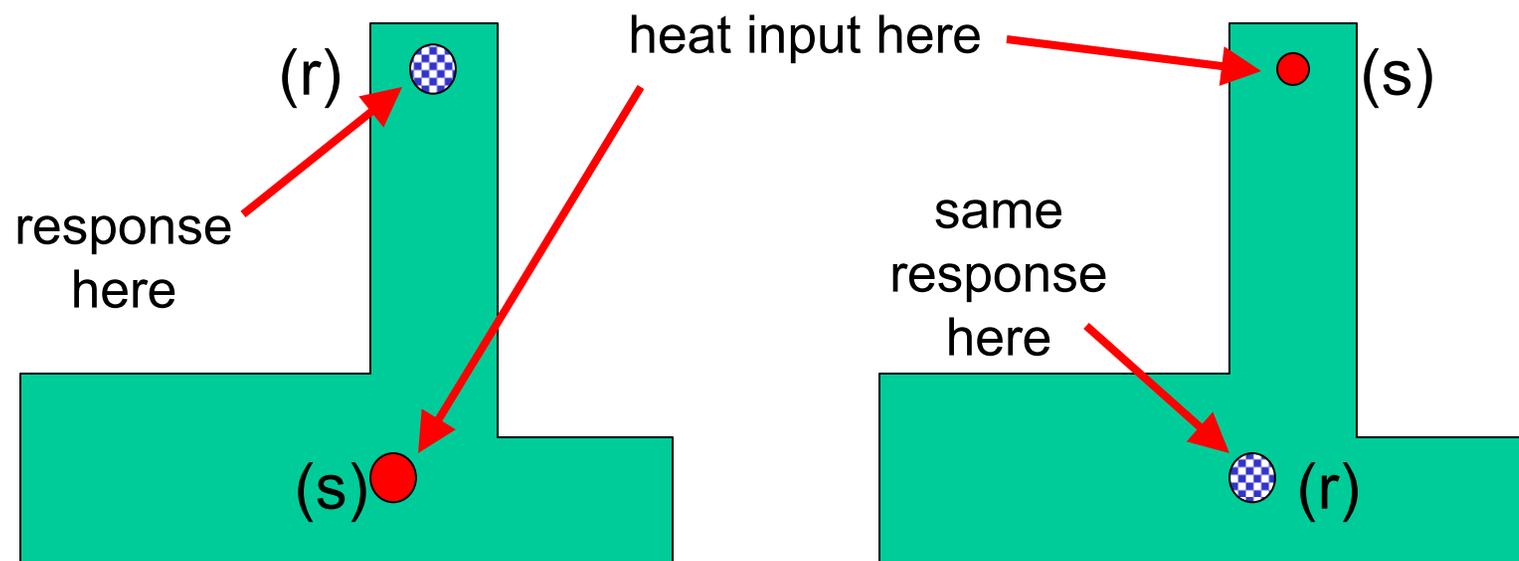
Electrical reciprocity



Thermal reciprocity



Another thermal reciprocity example



(square part of) matrix is symmetric

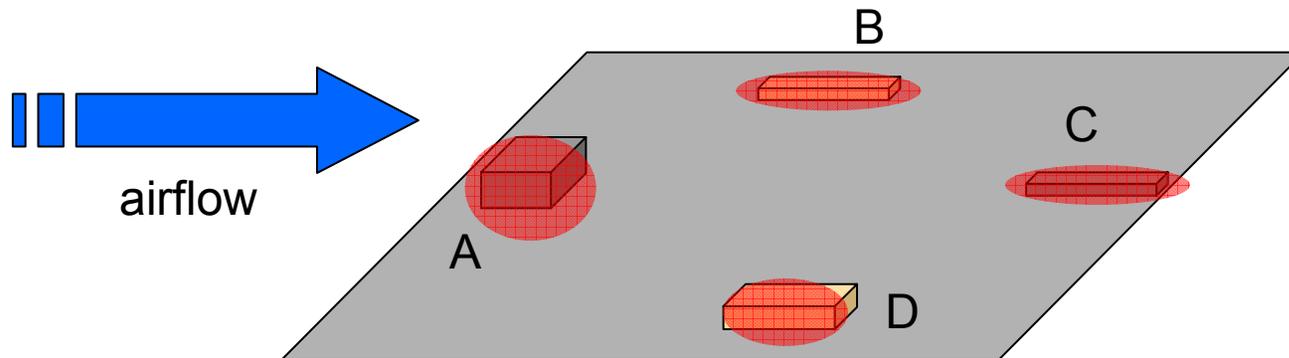
columns are the "x" heat sources

rows are
the "y"
response
locations

J1	75	65	55	60	22	10
J2	65	71	60	55	25	11
J3	55	60	65	61	21	15
J4	60	55	61	73	18	11
J5	22	25	21	18	125	14
J6	10	11	15	11	14	180
R1	73	65	55	59	22	10
R3	55	60	63	61	21	15
R5	20	24	14	19	95	15
B	65	63	62	63	21	12

When does reciprocity *NOT* Apply?

- Upwind and downwind in forced-convection dominated applications



Heat in at "A" will raise temperature of "C" more than heat in at "C" will raise temperature of "A"

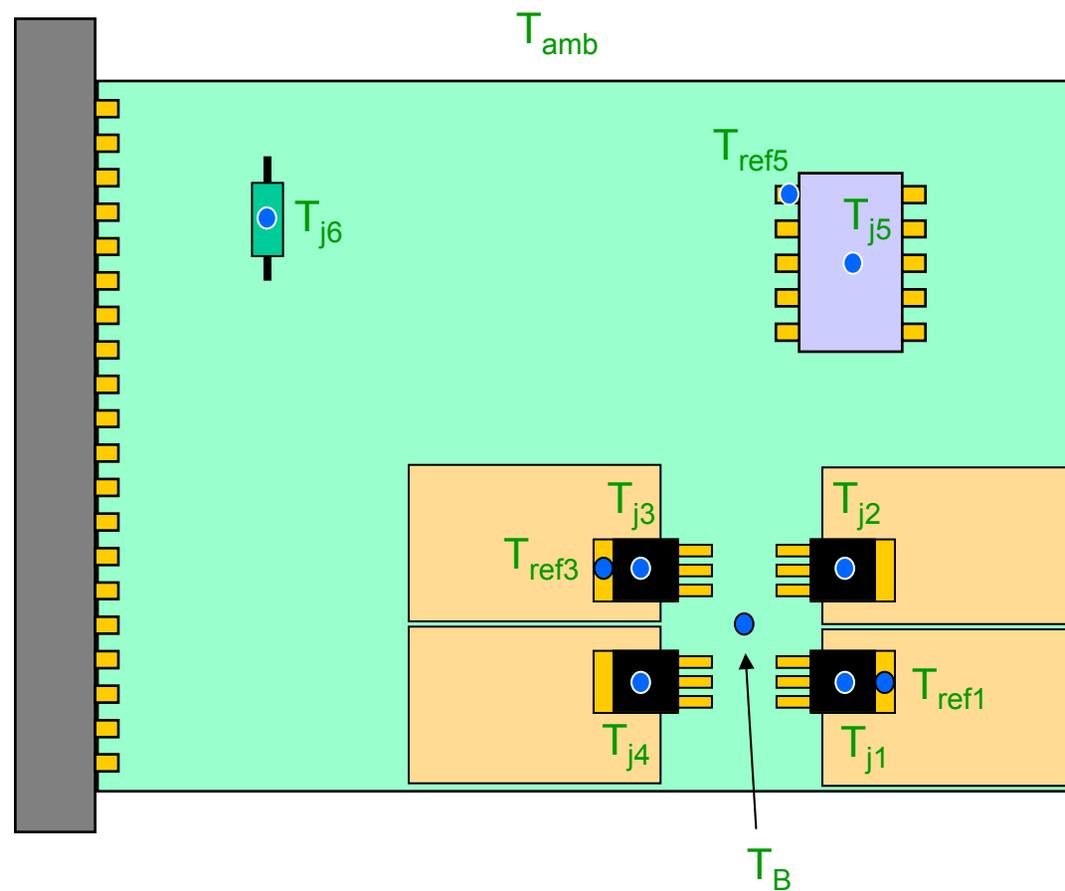
"B" and "D" may still be roughly reciprocal



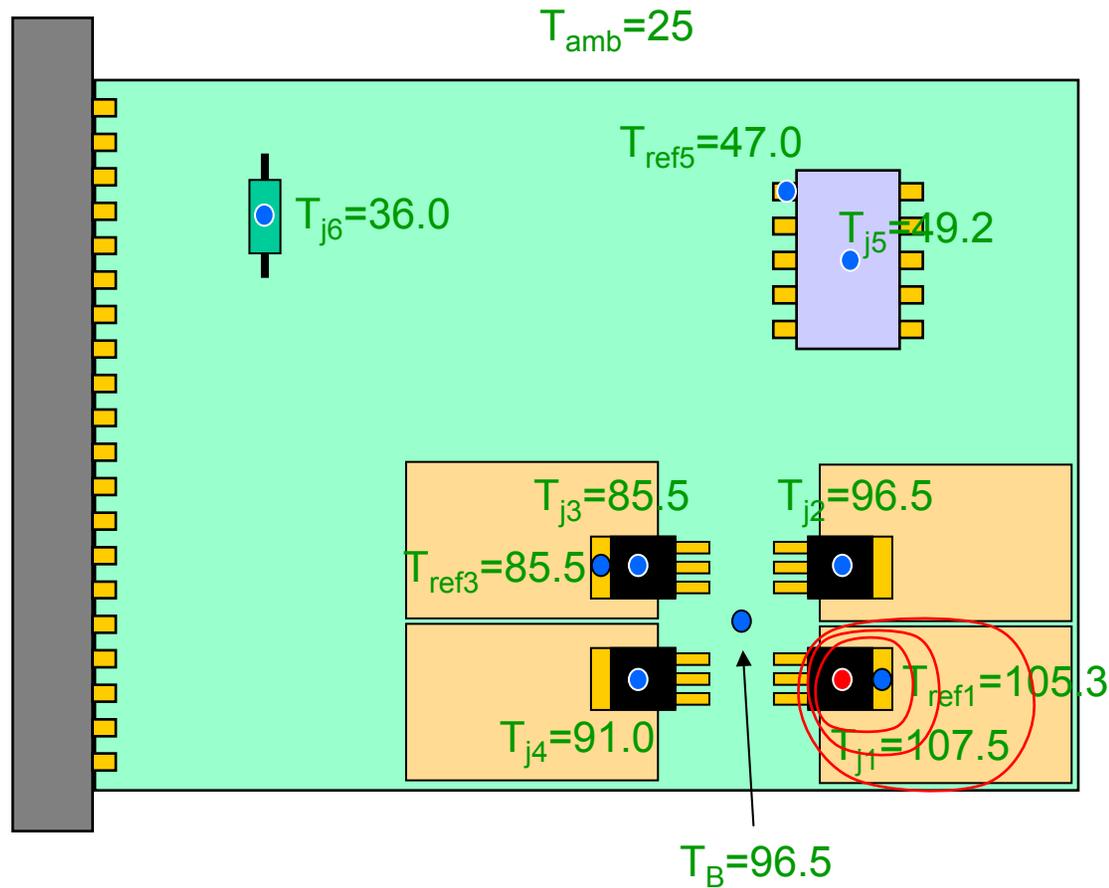
A linear superposition example

(unequivocal proof that a published
theta-JA is virtually meaningless)

Superposition example



Device 1 heated, 1.1 W



Reduce the data

$$\theta_{j1A} = \frac{T_{j1} - T_{amb}}{q_1} = \frac{107.5 - 25}{1.1} = 75$$

$$\Psi_{j2A} = \frac{T_{j2} - T_{amb}}{q_1} = \frac{96.5 - 25}{1.1} = 65$$

⋮

$$\Psi_{BA} = \frac{T_B - T_{amb}}{q_1} = \frac{96.5 - 25}{1.1} = 65$$

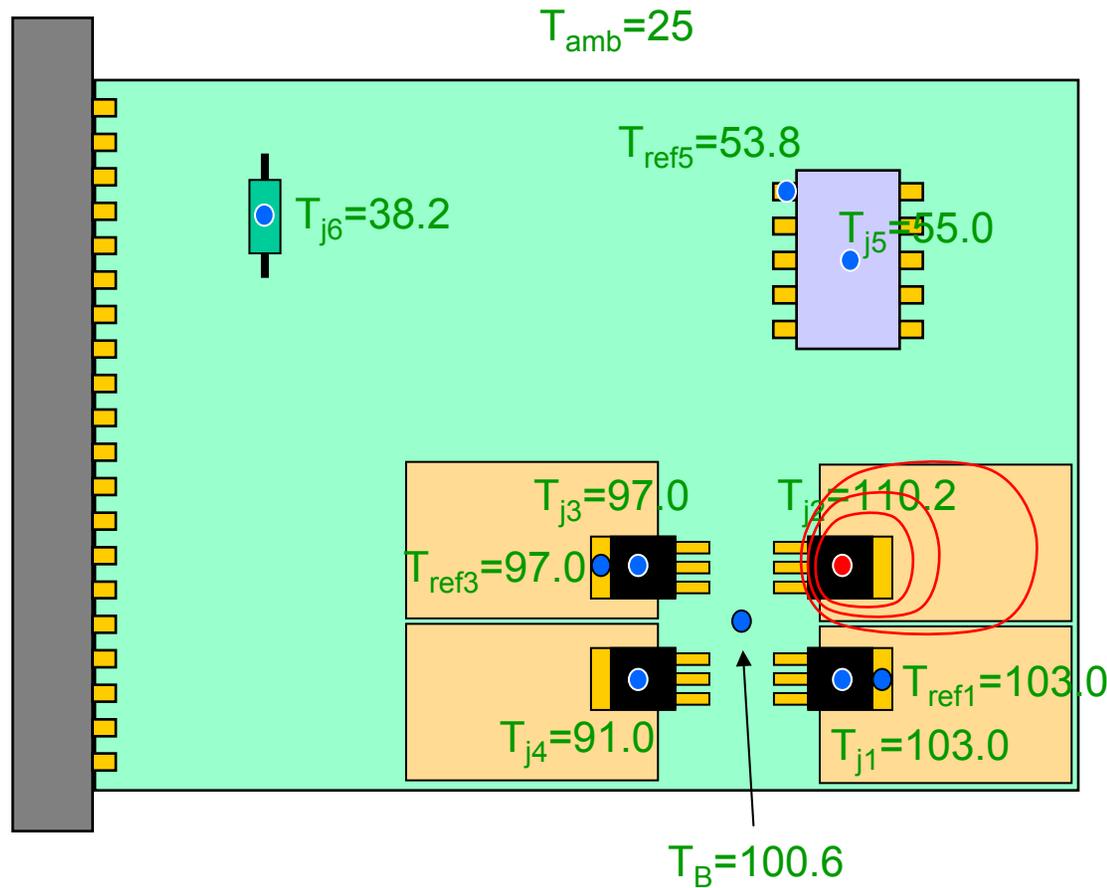
θ_{j1A}	75
Ψ_{j2A}	65
Ψ_{j3A}	55
Ψ_{j4A}	60
Ψ_{j5A}	22
Ψ_{j6A}	10
Ψ_{r1A}	73
Ψ_{r3A}	55
Ψ_{r5A}	20
Ψ_{BA}	65

Collect the θ/Ψ values in the matrix

J1	75					
J2	65					
J3	55					
J4	60					
J5	22					
J6	10					
R1	73					
R3	55					
R5	20					
B	65					



Device 2 heated, 1.2 W

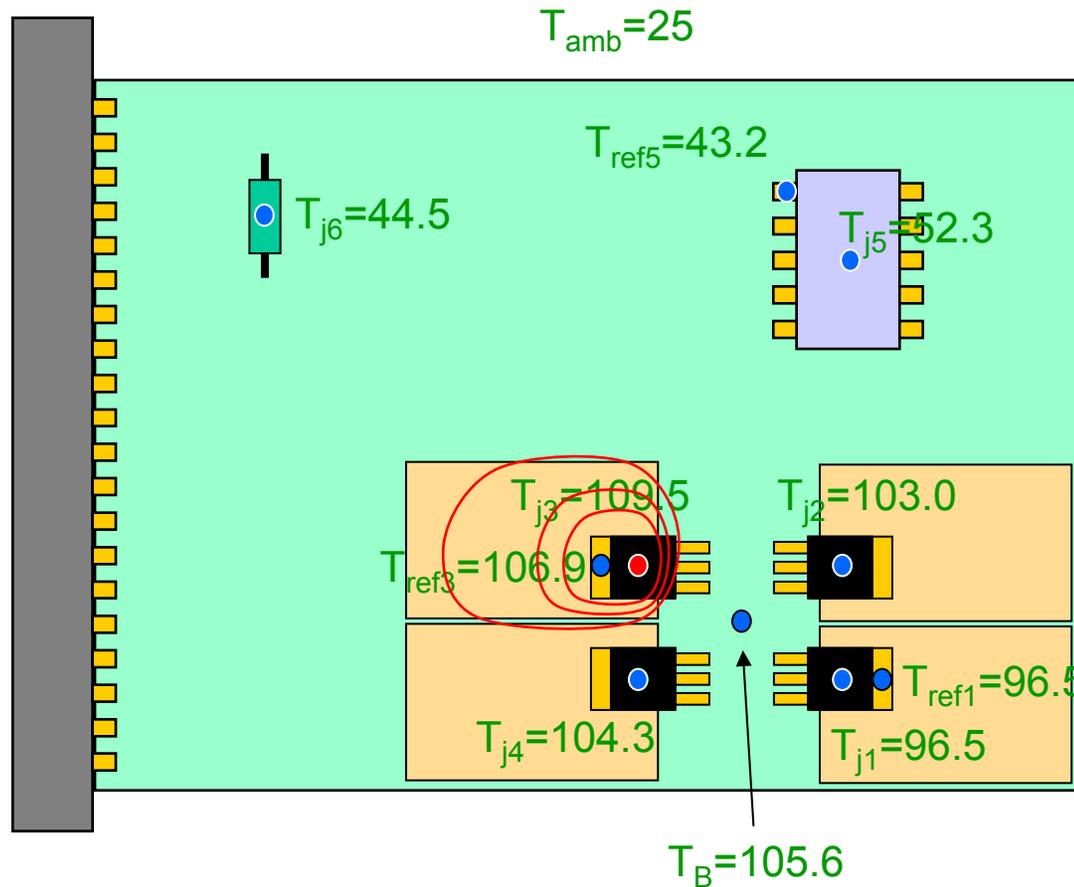


Ψ_{j1A}	65
θ_{j2A}	71
Ψ_{j3A}	60
Ψ_{j4A}	55
Ψ_{j5A}	25
Ψ_{j6A}	11
Ψ_{r1A}	65
Ψ_{r3A}	60
Ψ_{r5A}	24
Ψ_{BA}	63

Collect the θ/Ψ values

J1	75	65				
J2	65	71				
J3	55	60				
J4	60	55				
J5	22	25				
J6	10	11				
R1	73	65				
R3	55	60				
R5	20	24				
B	65	63				

Device 3 heated, 1.3 W

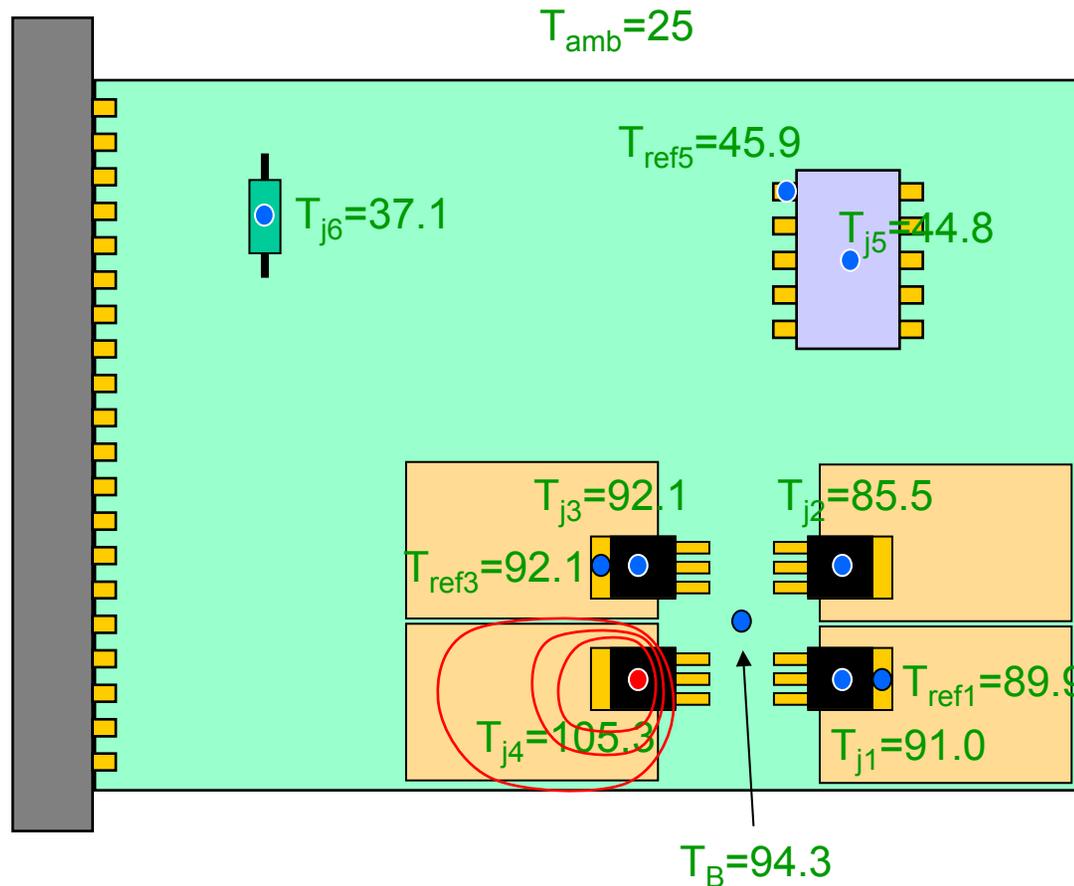


Ψ_{j1A}	55
Ψ_{j2A}	60
θ_{j3A}	65
Ψ_{j4A}	61
Ψ_{j5A}	21
Ψ_{j6A}	15
Ψ_{r1A}	55
Ψ_{r3A}	63
Ψ_{r5A}	14
Ψ_{BA}	62

Collect the θ/Ψ values

J1	75	65	55			
J2	65	71	60			
J3	55	60	65			
J4	60	55	61			
J5	22	25	21			
J6	10	11	15			
R1	73	65	55			
R3	55	60	63			
R5	20	24	14			
B	65	63	62			

Device 4 heated, 1.1 W

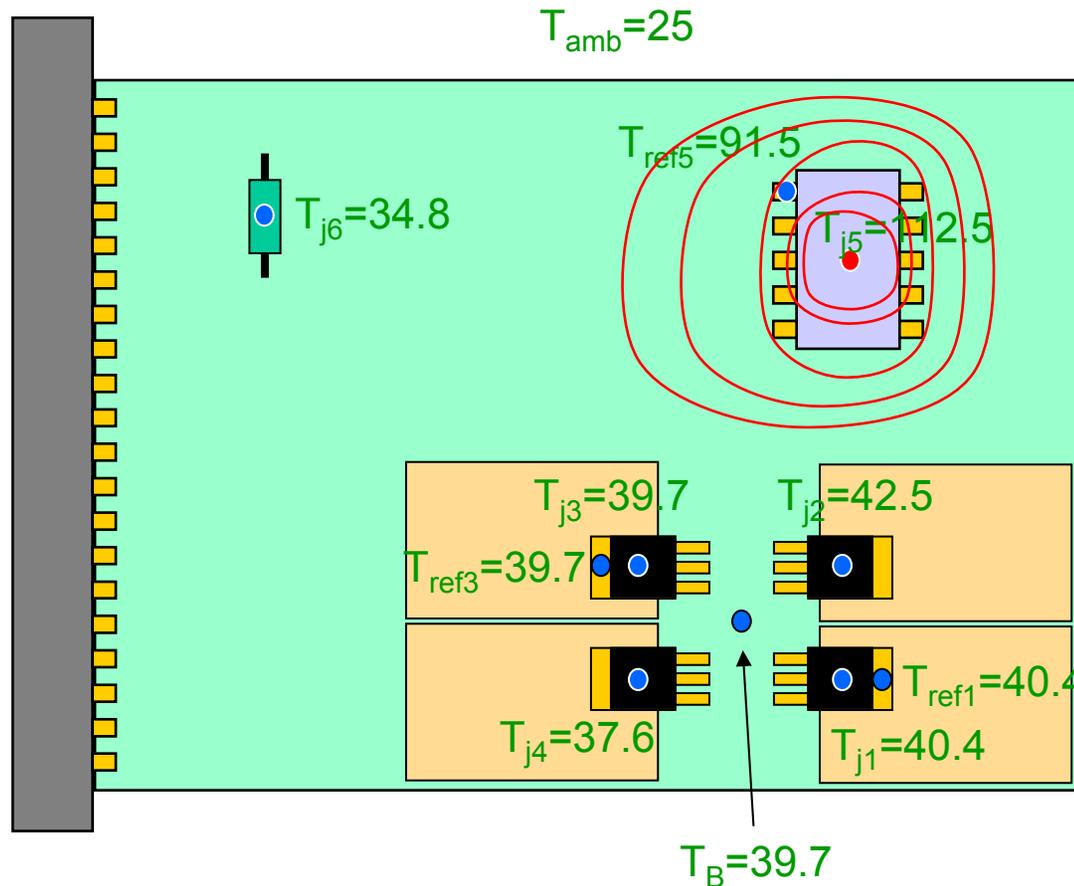


Ψ_{j1A}	60
Ψ_{j2A}	55
Ψ_{j3A}	61
θ_{j4A}	73
Ψ_{j5A}	18
Ψ_{j6A}	11
Ψ_{r1A}	59
Ψ_{r3A}	61
Ψ_{r5A}	19
Ψ_{BA}	63

Collect the θ/Ψ values

J1	75	65	55	60		
J2	65	71	60	55		
J3	55	60	65	61		
J4	60	55	61	73		
J5	22	25	21	18		
J6	10	11	15	11		
R1	73	65	55	59		
R3	55	60	63	61		
R5	20	24	14	19		
B	65	63	62	63		

Device 5 heated, 0.7 W

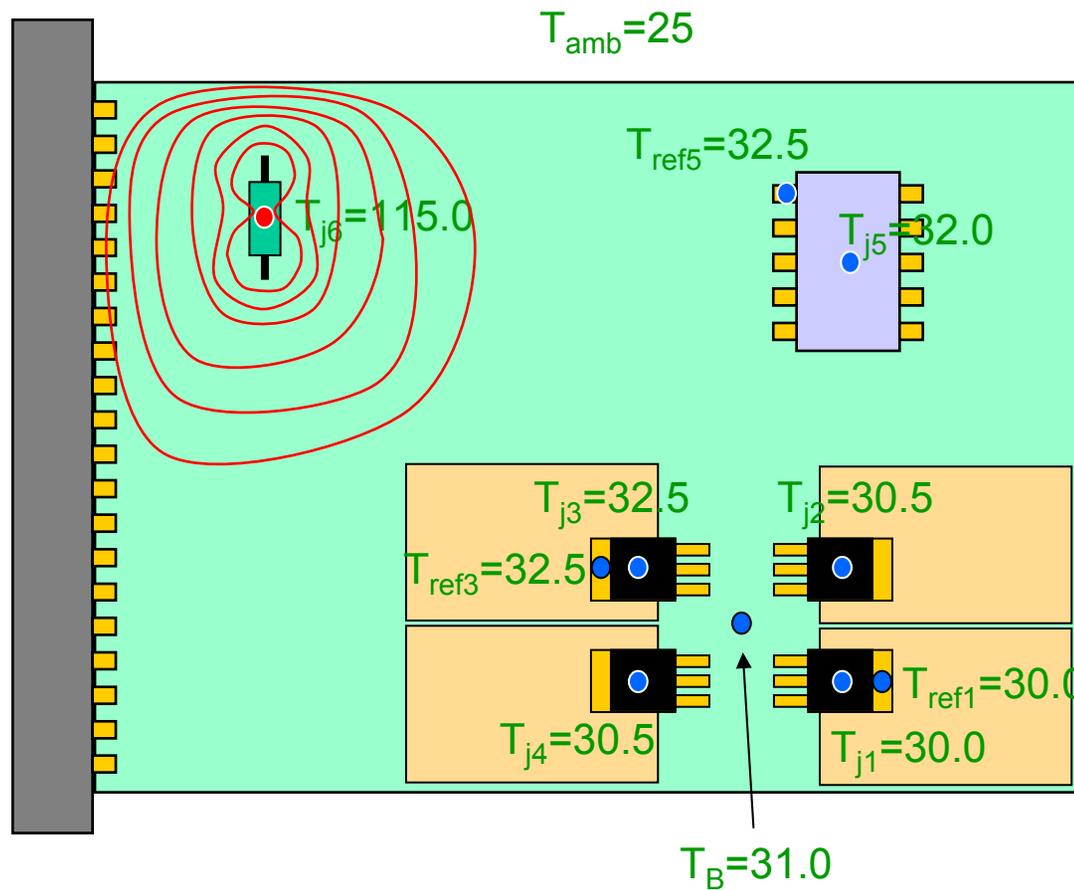


Ψ_{j1A}	22
Ψ_{j2A}	25
Ψ_{j3A}	21
Ψ_{j4A}	18
θ_{j5A}	125
Ψ_{j6A}	14
Ψ_{r1A}	22
Ψ_{r3A}	21
Ψ_{r5A}	95
Ψ_{BA}	21

Collect the θ/Ψ values

J1	75	65	55	60	22	
J2	65	71	60	55	25	
J3	55	60	65	61	21	
J4	60	55	61	73	18	
J5	22	25	21	18	125	
J6	10	11	15	11	14	
R1	73	65	55	59	22	
R3	55	60	63	61	21	
R5	20	24	14	19	95	
B	65	63	62	63	21	

Device 6 heated, 0.5 W

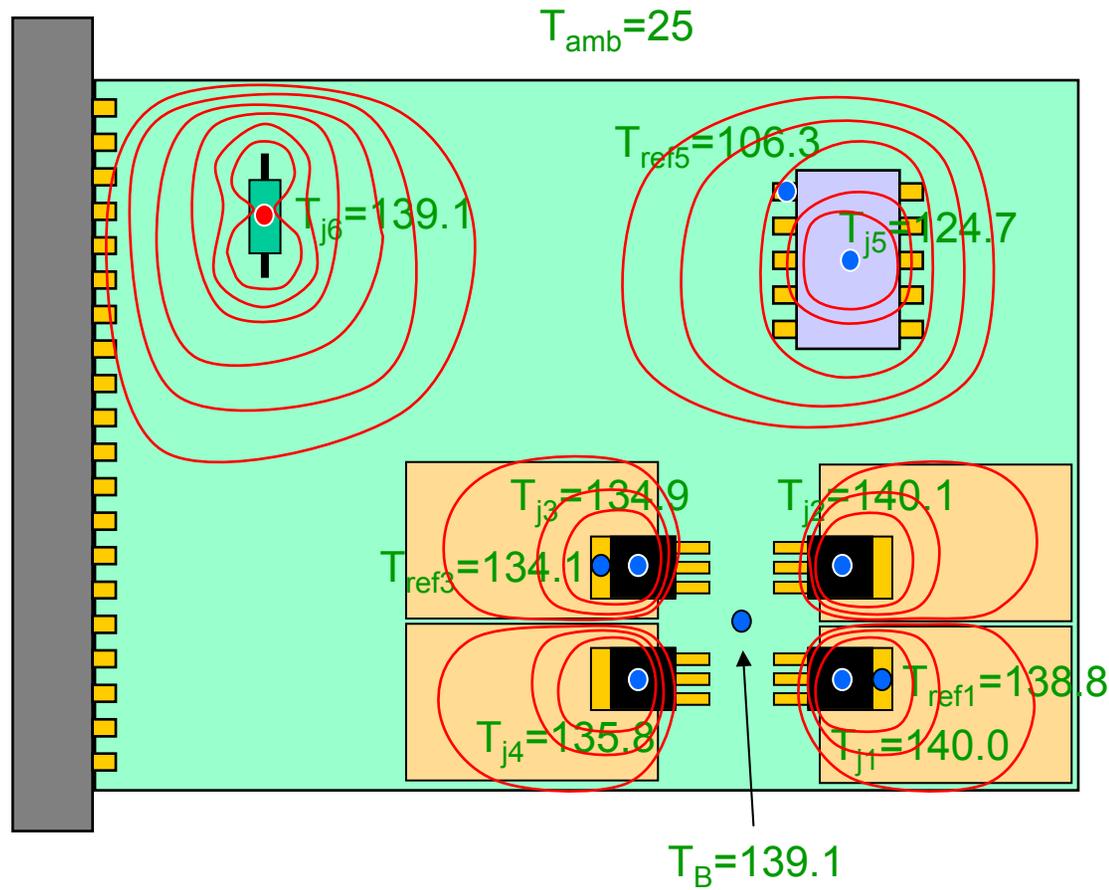


Ψ_{j1A}	10
Ψ_{j2A}	11
Ψ_{j3A}	15
Ψ_{j4A}	11
Ψ_{j5A}	14
θ_{j6A}	180
Ψ_{r1A}	10
Ψ_{r3A}	15
Ψ_{r5A}	15
Ψ_{BA}	12

Collect the θ/Ψ values

J1	75	65	55	60	22	10
J2	65	71	60	55	25	11
J3	55	60	65	61	21	15
J4	60	55	61	73	18	11
J5	22	25	21	18	125	14
J6	10	11	15	11	14	180
R1	73	65	55	59	22	10
R3	55	60	63	61	21	15
R5	20	24	14	19	95	15
B	65	63	62	63	21	12

Now apply actual power



Actual power in application

q_{j1}	.4
q_{j2}	.4
q_{j3}	.4
q_{j4}	.4
q_{j5}	.5
q_{j6}	.2

Compute some effective θ/Ψ values

Take T_{j1} , for instance. Remember when it was heated all alone, we calculated its self-heating theta-JA like this:

$$\theta_{j1A} = \frac{T_{j1} - T_{amb}}{q_1}$$

Now let's see:

$$\theta_{j1A} = \frac{T_{j1} - T_{amb}}{q_1} = \frac{140 - 25}{0.4} = 288 \neq$$

And that's not just a single aberration!

Self heating		
θ_{j1A}	288	← 3.8x - 75
θ_{j2A}	288	← 4.1x - 71
θ_{j3A}	274	← 4.2x - 65
θ_{j4A}	277	← 3.8x - 73
θ_{j5A}	199	← 1.6x - 125
θ_{j6A}	309	← 1.7x - 180

Junction to Reference		
Ψ_{j1-R1}	3.0	← 1.5x - 2.0
Ψ_{j3-R3}	2.0	← 1.0x - 2.0
Ψ_{j5-R5}	36.8	← 1.2x - 30.0

Junction to Board		
Ψ_{j1-B}	2.2	← 0.2x - 10.0
Ψ_{j2-B}	2.5	← 0.3x - 8.0
Ψ_{j3-B}	-10.5	← -3.5x - 3.0
Ψ_{j4-B}	-8.3	← -0.8x - 10.0

Is the moral clear?

- You simply *cannot* use published theta-JA values for devices in your real system, even if those values are perfectly accurate and correct as reported on the datasheet and you know the exact specifications of the test conditions.
- Not unless your actual application is identical to the manufacturer's test board – and uses just that one device *all by itself*.

So is it *really* this bad?

Only sort-of. Let's revisit the math for one device ...

$$\begin{Bmatrix} T_{j1} \\ T_{j2} \\ \vdots \\ T_{jn} \end{Bmatrix} = \begin{bmatrix} \theta_{J1A} & \Psi_{12} & \dots & \Psi_{1n} \\ \Psi_{12} & \theta_{J2A} & & \Psi_{2n} \\ \vdots & & \ddots & \vdots \\ \Psi_{1n} & \Psi_{2n} & \dots & \theta_{JnA} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix} + T_a$$

$$T_{j1} = \theta_{J1A} q_1 + \Psi_{12} q_2 + \dots + \Psi_{1n} q_n + T_a$$

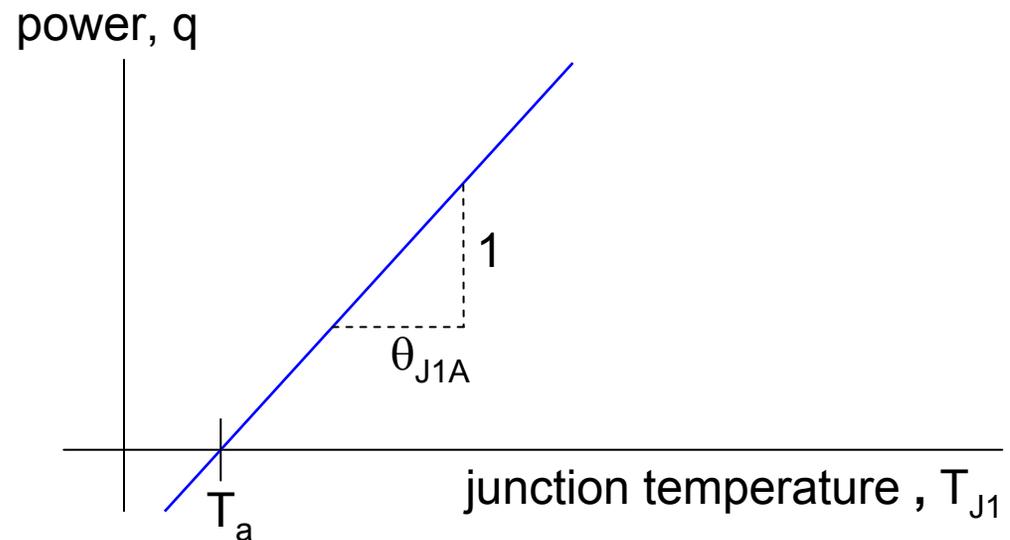
$$T_{j1} = \theta_{J1A} q_1 + \sum_{n=2}^n \Psi_{1n} q_n + T_a$$

“effective” ambient

A graphical view

Isolated device

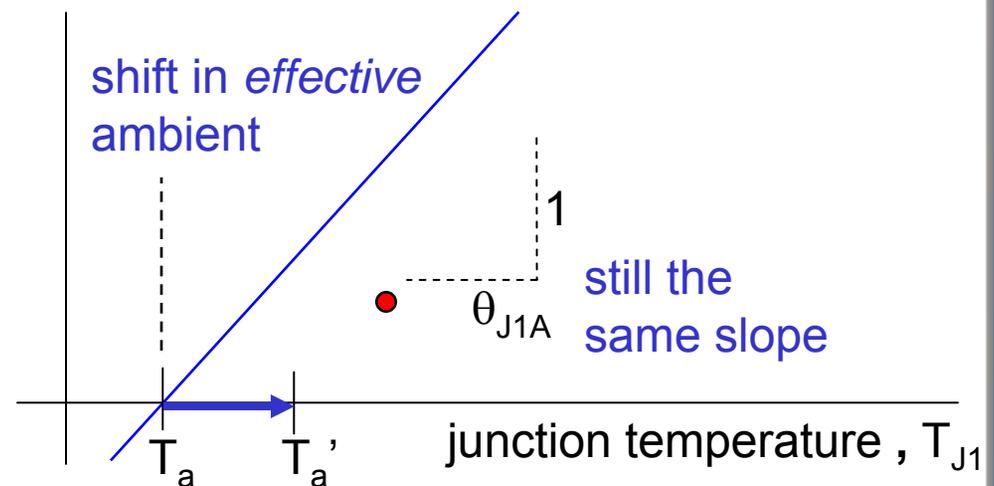
$$T_{j1} = \theta_{J1A} q_1 + T_a$$



Device in a system

$$T_{j1} = \theta_{J1A} q_1 + \underbrace{\sum_2^n \Psi_{1n} q_n}_{T_a'} + T_a$$

$$= \theta_{J1A} q_1 + T_a'$$





How does *effective ambient* relate to board temperature?

“system” slope for isolated device

if any of *these* are non-zero, T'_a will be higher than T_a

$$\begin{aligned}
 T_{j1} &= \theta_{j1a} \cdot Q_1 + \sum_{i=2}^n (\Psi_{i1} \cdot Q_i) + T_a \\
 &= (\theta_{j1B} + \theta_{B1a}) \cdot Q_1 + T_a \\
 &= \theta_{j1B} \cdot Q_1 + \theta_{B1a} \cdot Q_1 + T_a \\
 &= \Delta T_{j1B} + \Delta T_{B1a} + T_a
 \end{aligned}$$

temperature rise, board to J1
temperature rise, ambient to board

effective ambient

when Q_1 is zero, T_a is the effective ambient temperature. If Q_1 is non-zero, T_a will be higher than the actual ambient temperature.

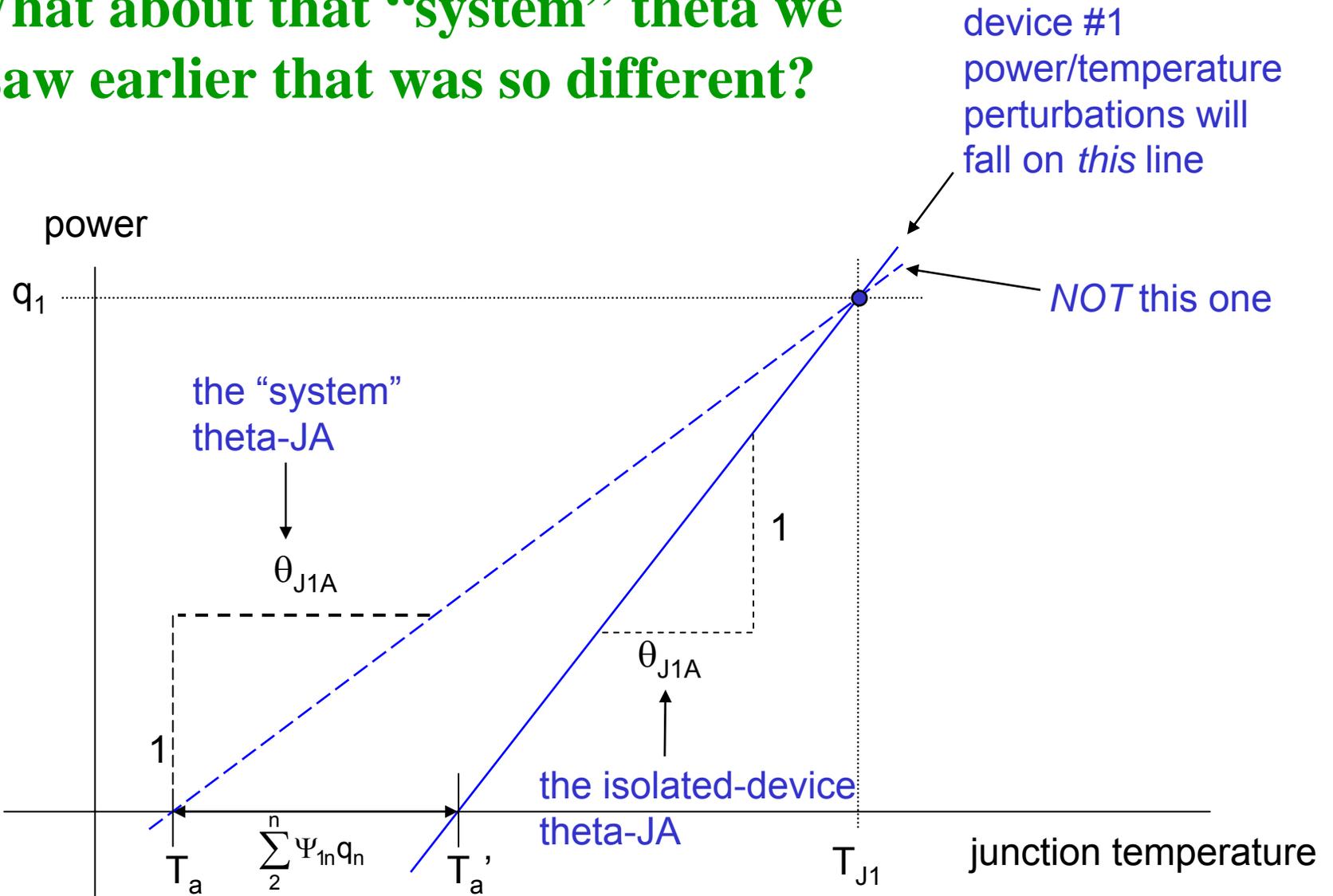


**How does *effective ambient* relate
to local air temperature?**

NOT.



What about that “system” theta we saw earlier that was so different?





System modeling

Filling in the theta-matrix

- Handy formulas for quick estimates
- Utilizing symmetry

Conduction resistance

basic heat transfer relationship for 1-D conduction

$$q = k \cdot A \cdot \frac{dT}{dx} \approx k \cdot A \cdot \frac{\Delta T}{L}$$

if we define

$$R = \frac{\Delta T}{q}$$

then

$$R = \frac{L}{k \cdot A}$$

Convection resistance

basic heat transfer relationship for surface cooling

$$q = h \cdot A \cdot \Delta T$$

if we define

$$R = \frac{\Delta T}{q}$$

then

$$R = \frac{1}{hA}$$

Radiation resistance

basic heat transfer relationship for surface radiation

$$\begin{aligned}
 q &= \sigma \cdot \varepsilon \cdot F \cdot A \cdot (T^4 - T_a^4) \\
 &= \sigma \cdot \varepsilon \cdot F \cdot A \cdot (T^2 + T_a^2)(T + T_a)(T - T_a) \\
 &= \sigma \cdot \varepsilon \cdot F \cdot A \cdot (T^2 + T_a^2)(T + T_a)\Delta T
 \end{aligned}$$

if we define

$$R = \frac{\Delta T}{q}$$

then

$$R = \frac{1}{\sigma \varepsilon F A (T^2 + T_a^2)(T + T_a)}$$

temperatures *must*
be expressed in
degrees “absolute”!

Thermal capacitance and time constant

capacitance is ability to store energy
specific heat is energy storage/mass

$$C = \rho c_p V$$

so if

$$R = \frac{L}{k \cdot A} \quad \text{and} \quad C = \rho c_p (L \cdot A)$$

then

$$\tau = \frac{\rho c_p L^2}{k} = \frac{L^2}{\alpha}$$

based on simple RC concept,
relate rate of storage to rate of flux
result is

$$\tau = RC$$

and if

$$R = \frac{1}{h \cdot A} \quad \text{and} \quad C = \rho c_p (L \cdot A)$$

then

$$\tau = \frac{\rho c_p L}{h}$$

Some useful formulas

- conduction resistance..... $R = \frac{L}{k \cdot A}$
- convection resistance..... $R = \frac{1}{h \cdot A}$
- thermal capacitance..... $C = \rho c_p V$
- characteristic time..... $\tau = \frac{L^2}{\alpha}$
 - (dominated by 1-D conduction)
- characteristic time..... $\tau = \frac{\rho c_p L}{h}$
 - (dominated by 1-D convection)
- short-time 1-D transient response..... $\Delta T = \frac{2}{\sqrt{\pi}} \frac{Q}{A \eta} \sqrt{t}$

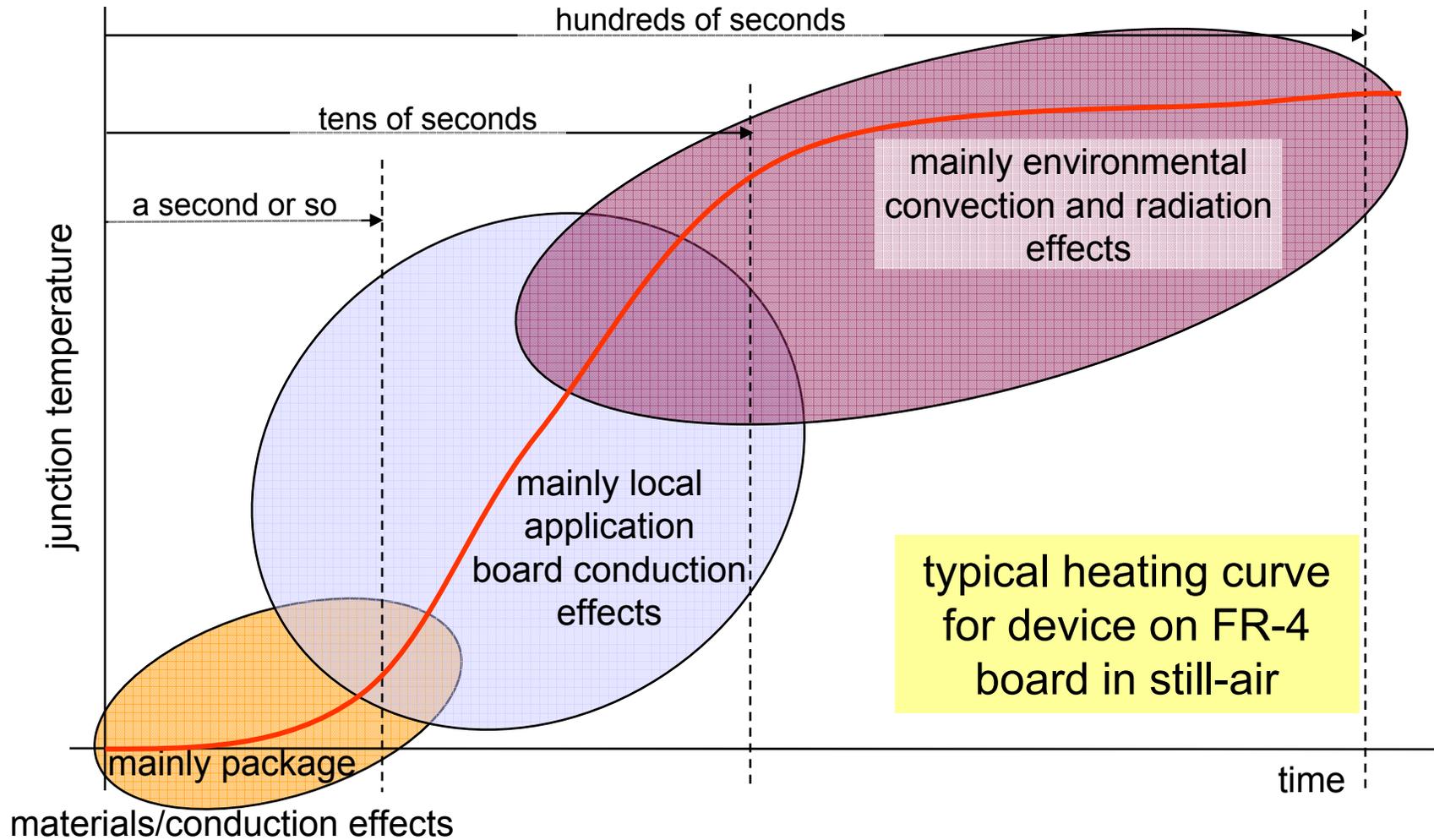
Terms used in preceding formulas

- L - thermal path length
- A - thermal path cross-sectional area
- k - thermal conductivity
- ρ - density
- c_p - heat capacity
- V - volume of material (L·A)
- α - thermal diffusivity
- η - thermal effusivity
- h - convection heat-transfer “film coefficient”)
- ΔT - junction temperature rise
- Q - heating power
- t - time since heat was first applied

$$\alpha = \frac{k}{\rho c_p}$$

$$\eta = \sqrt{\rho c_p k}$$

When do these effects enter?

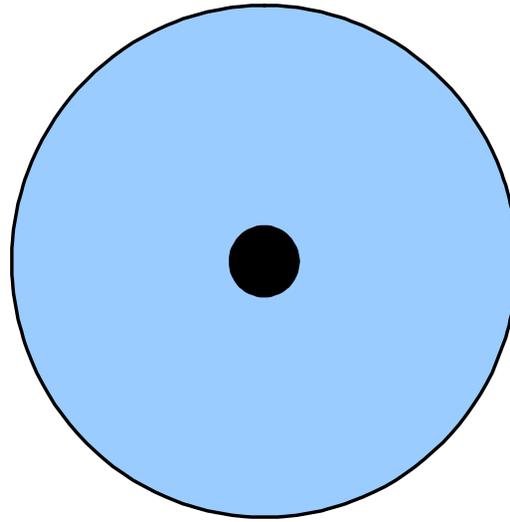




Utilize symmetry whenever possible

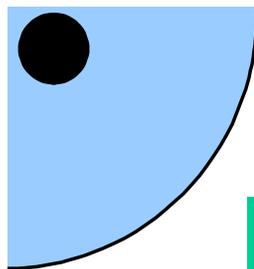


if



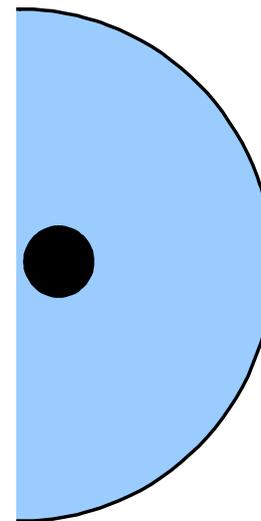
$$\Rightarrow R$$

then



$$\approx 4R$$

and



$$\approx 2R$$

Cylindrical and spherical conduction (through radial thickness) resistance formulas

Half-cylinder	$R = \frac{\ln\left(\frac{r_o}{r_i}\right)}{\pi \cdot k \cdot L}$	Hemisphere
	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{2\pi \cdot k}$	
	[included angle]	[solid angle]
Full cylinder	$R = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi \cdot k \cdot L}$	Full sphere
	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi \cdot k}$	

- where
- L – cylinder length
 - r_i – inner radius
 - r_o – outer radius



Predicting the temperature of high power components

- The device and system are equally important to get right

Using the previous board example ...

theta array

J1	75	65	55	60	22	10
J2	65	71	60	55	25	11
J3	55	60	65	61	21	15
J4	60	55	61	73	18	11
J5	22	25	21	18	125	14
J6	10	11	15	11	14	180
R1	73	65	55	59	22	10
R3	55	60	63	61	21	15
R5	20	24	14	19	95	15
B	65	63	62	63	21	12

power
vector

q_{j1}	0.5
q_{j2}	0.5
q_{j3}	0.5
q_{j4}	0.5
q_{j5}	0.2
q_{j6}	0.02

Observe the relative contributions

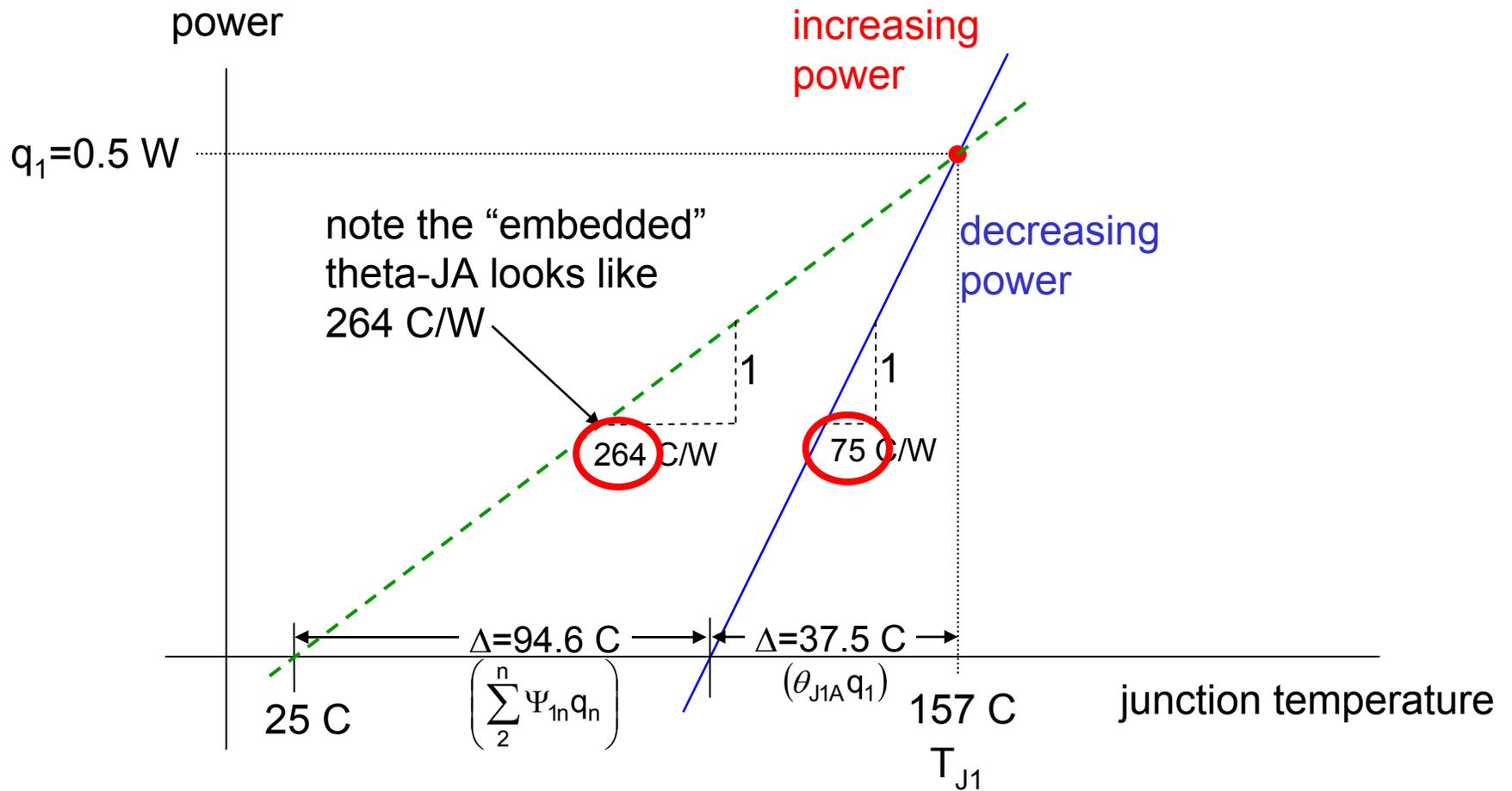
For junction 1 (a high power component) we have:

the device itself ...

the other devices ...

$$\begin{aligned}
 &= (75 \times 0.5) + (65 \times 0.5) + (55 \times 0.5) + (60 \times 0.5) + (22 \times 0.2) + (10 \times 0.02) + 25 \\
 &= 37.5 + 32.5 + 27.5 + 30 + 4.4 + 0.2 + 25 \\
 &= 37.5 + 94.6 + 25
 \end{aligned}$$

Graphically, it looks like this:





Predicting the temperature of low power components

- The system is probably more important than the device

Using the previous board example ...

theta array

J1	75	65	55	60	22	10
J2	65	71	60	55	25	11
J3	55	60	65	61	21	15
J4	60	55	61	73	18	11
J5	22	25	21	18	125	14
J6	10	11	15	11	14	180
R1	73	65	55	59	22	10
R3	55	60	63	61	21	15
R5	20	24	14	19	95	15
B	65	63	62	63	21	12

power
vector

q_{j1}	0.5
q_{j2}	0.5
q_{j3}	0.5
q_{j4}	0.5
q_{j5}	0.2
q_{j6}	0.02

Relative contributions to ΔT_{J6}

the other devices ...

$$= (10 \times 0.5) + (11 \times 0.5) + (15 \times 0.5) + (11 \times 0.5) + (14 \times 0.2) + (180 \times 0.02)$$

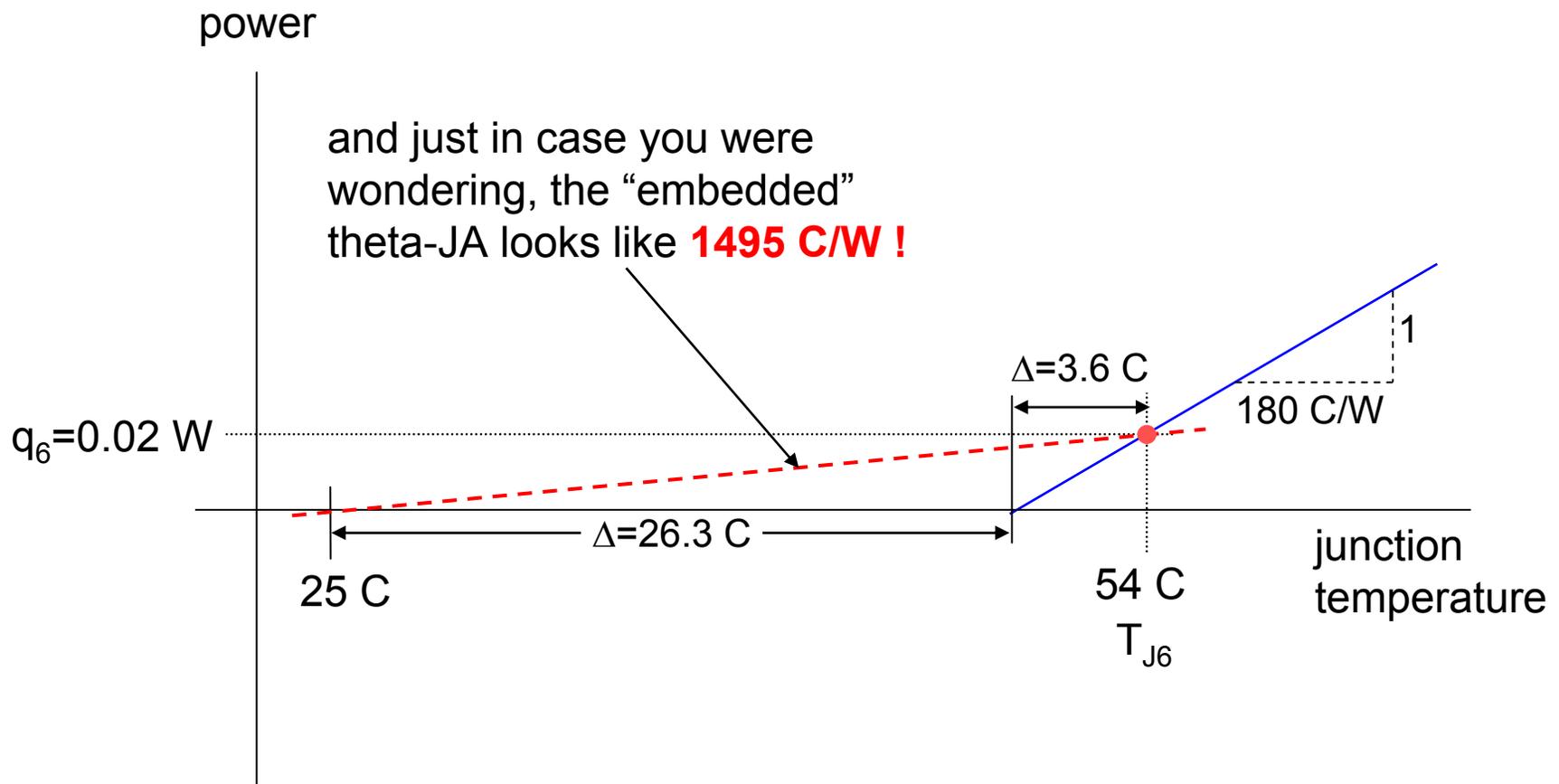
the device itself ...

+ 25

$$= 5.0 + 5.5 + 7.5 + 5.5 + 2.8 + 3.6 + 25$$

$$= 26.3 + 3.6 + 25$$

Graphically, low-power device #6 looks like this:





Controlling the matrix

How to harness this math in Excel®



3x3 theta matrix, 3x1 power vector Excel® math

{=array formula notation} obtained by using *Ctrl-Shift-Enter* rather than ordinary *Enter* **Matrix MULTi**ply multi-cell placement of array formula

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
18	Non-symmetric three junction device (note matrix itself is still symmetric around main diagonal)										
19						power		ambient	resulting temperatures		
20	Tj1	100	75	65		1.1		25	309.5		Tj1
21	Tj2	75	95	70		1.2			312.5		Tj2
22	Tj3	65	70	90		1.3			297.5		Tj3

The formula bar shows: `{=MMULT(B20:D22,F20:F22)+H20}`

theta matrix

power vector

array reference to theta matrix

array reference to power vector



7x3 theta matrix, 3x1 power vector Excel® math

theta matrix is no longer square – don't forget to use
 # of columns still must equal *Ctrl-Shift-Enter*
 # of rows of power vector to invoke array formula notation

array formula now occupies 7 cells

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J
25	Non-symmetric three junction device with a couple of additional reference temperatures									
26						power				resulting temperatures
27	Tj1	100	75	65		1.1				Tj1
28	Tj2	75	95	70		1.2				Tj2
29	Tj3	65	78	90		1.3				Tj3
30	case_1	64	50	45						case_1
31	case_2	50	64	38		ambient				case_2
32	case_3	42	50	68		25				case_3
33	board-cent	30	35	30						board-cent

The formula bar shows: `{=MMULT(B27:D33,F27:F29)+F32}`

The resulting temperatures are displayed in cells J27:J33:

309.5
312.5
297.5
213.9
206.2
219.6
139.0



7x3 theta matrix, 3x2 power vector Excel® math

power “vector” is now a 3x2 array – each column is a different power scenario, yet both are still processed using a single array (MMULT) formula

the single MMULT array formula now occupies 7 rows and 2 columns (one column for each independent power scenario result)

Microsoft Excel - theta matrix examples.xls

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

100% Arial 10 B I U

140 {=MMULT(B38:D44,F38:G40)+F43}

	A	B	C	D	E	F	G	H	I	J	K
36	Non-symmetric three junction device, additional ref temps, and multiple power vectors										
37					vect1	vect2	resulting temperatures				
38	Tj1	100	75	65	1.1	0.5			309.5	140.0	Tj1
39	Tj2	75	95	70	1.2	0			312.5	132.5	Tj2
40	Tj3	65	70	90	1.3	1			297.5	147.5	Tj3
41	case_1	64	50	45					213.9	102.0	case_1
42	case_2	50	64	38	ambient				206.2	88.0	case_2
43	case_3	42	50	68	25				219.6	114.0	case_3
44	board-cent	30	35	30					139.0	70.0	board-cent

example formulas

Ready NUM

Package-shrink “gotcha”

Often, much or even *most* of theta-JA depends on what *isn't* the package?

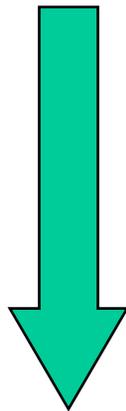
For instance, what if your cooling depends significantly on convection from the board surface (whether free or forced air)?

$$q = h \cdot A \cdot \Delta T \quad \Leftrightarrow \quad A = \frac{q}{h \cdot \Delta T}$$

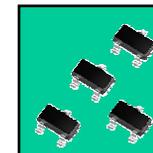
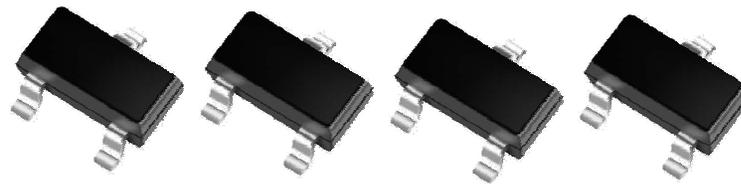
So never mind the *package* resistance, the *board* can only transfer a certain amount of heat to the air:

Heat transfer 101

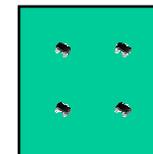
Decrease size but not power dissipation



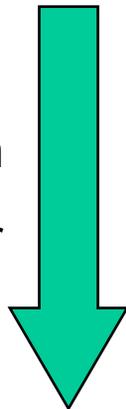
SOT23



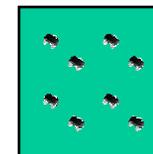
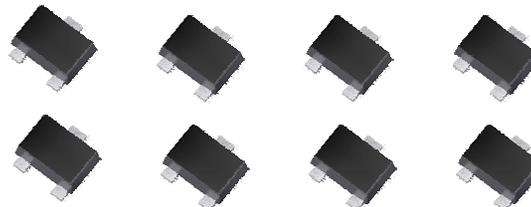
SOT723



Decrease size **and** reduce power dissipation (R_{DS(on)} or other electrical performance)



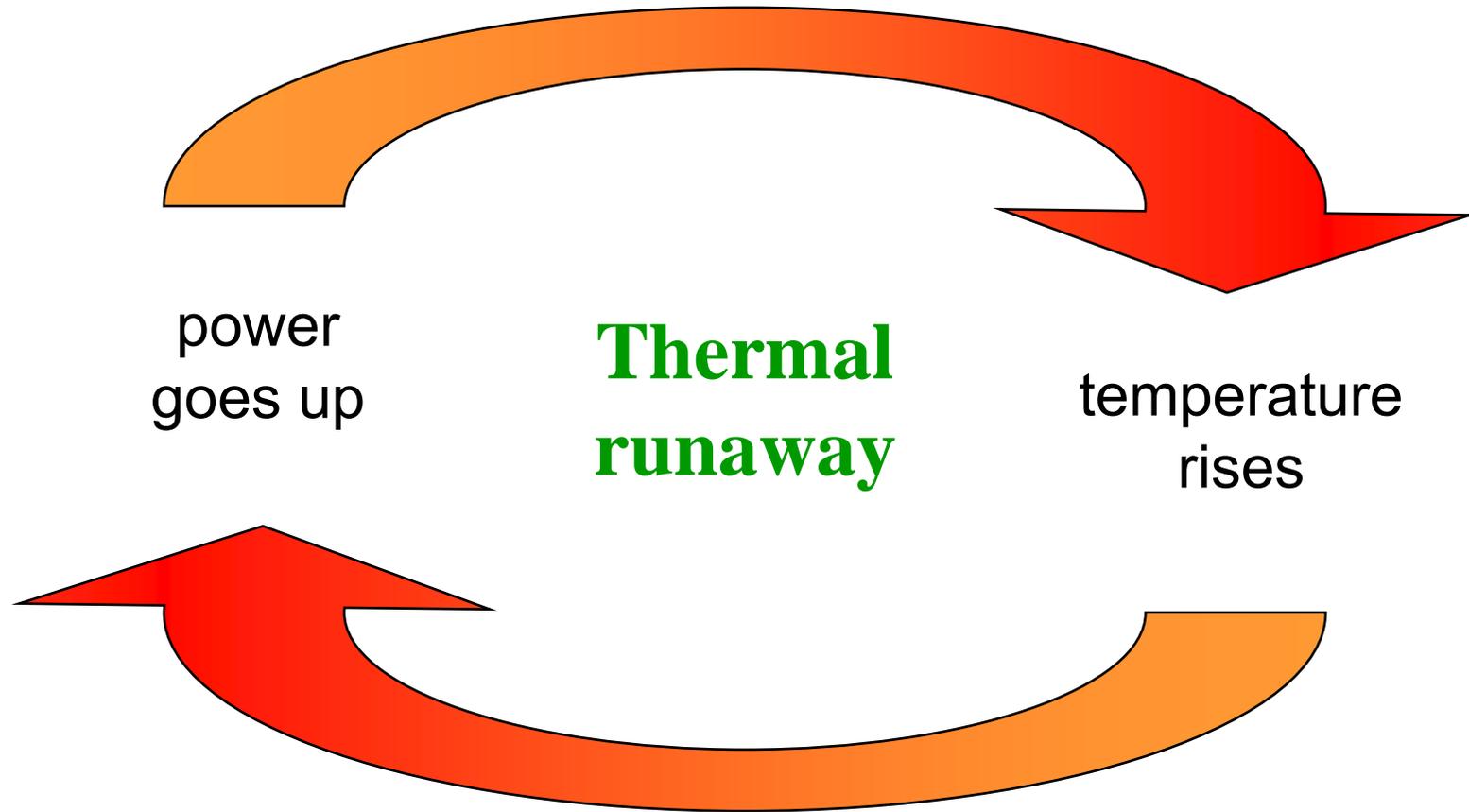
SOT723 @ **0.125 W**, $\Delta T = 100^{\circ}\text{C}$, **8** packages per 1000 mm²



Thermal runaway

- Theory
 - What is it?
 - When can it happen?
 - A mathematical model of power-law runaway
- An actual device example
- The surrounding system
 - A paradox and its resolution
 - how other components in a complete system affect runaway in a susceptible device
- Review

typical thermal response



power goes up

Thermal runaway

temperature rises

nonlinear electrical response

Thermal runaway

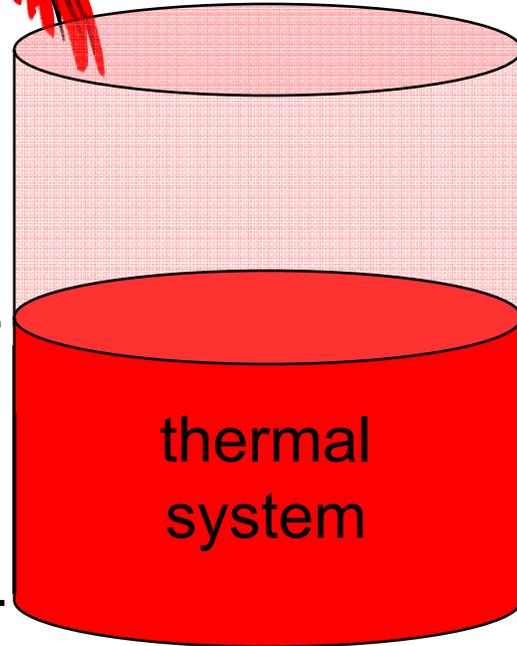
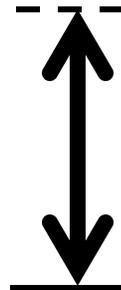
- System thermal resistance isn't low enough to shed small perturbations of power
- Nonlinear power vs. junction temperature device characteristic

Balance of power

input
power IN
power
increases

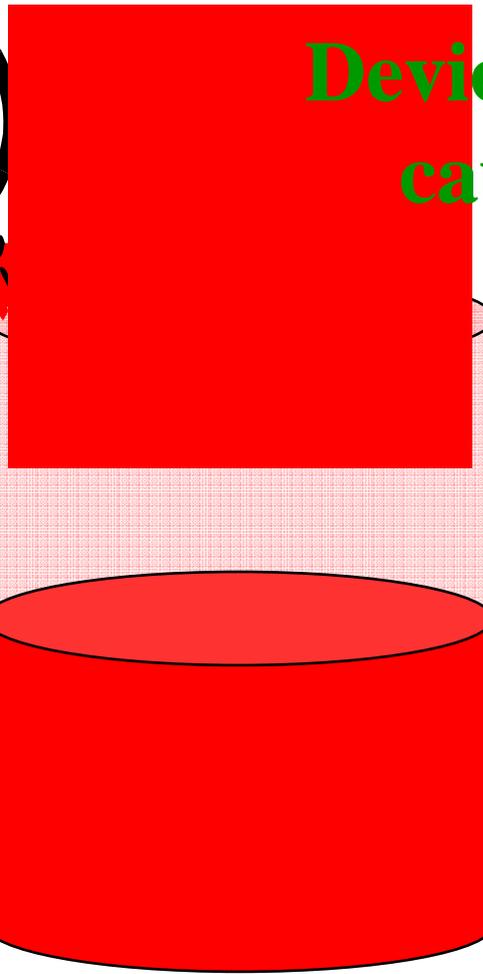


temperature
~~temperature~~
increases
is fixed



power
equals
dissipation
power OUT
rises

Device nonlinearity causes trouble



By design,
power is balanced
and temperature
increases
temperature is fixed.

power
dissipation
rises

A linear thermal cooling system

$$T_J = Q \cdot \theta_{Jx} + T_x$$

junction temperature as function of
power, theta, and ground

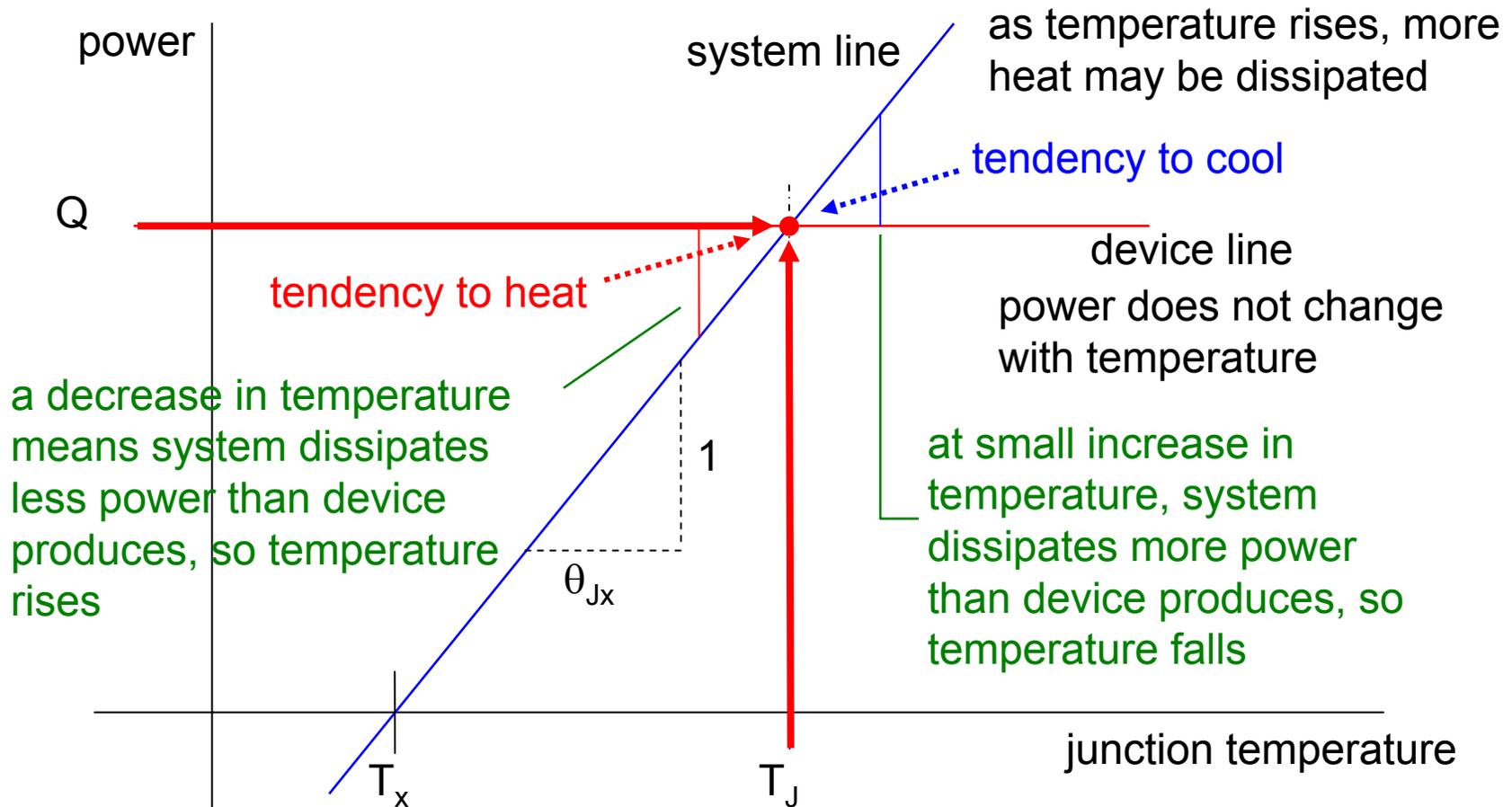
$$Q = \frac{T_J - T_x}{\theta_{Jx}}$$

... solving for power

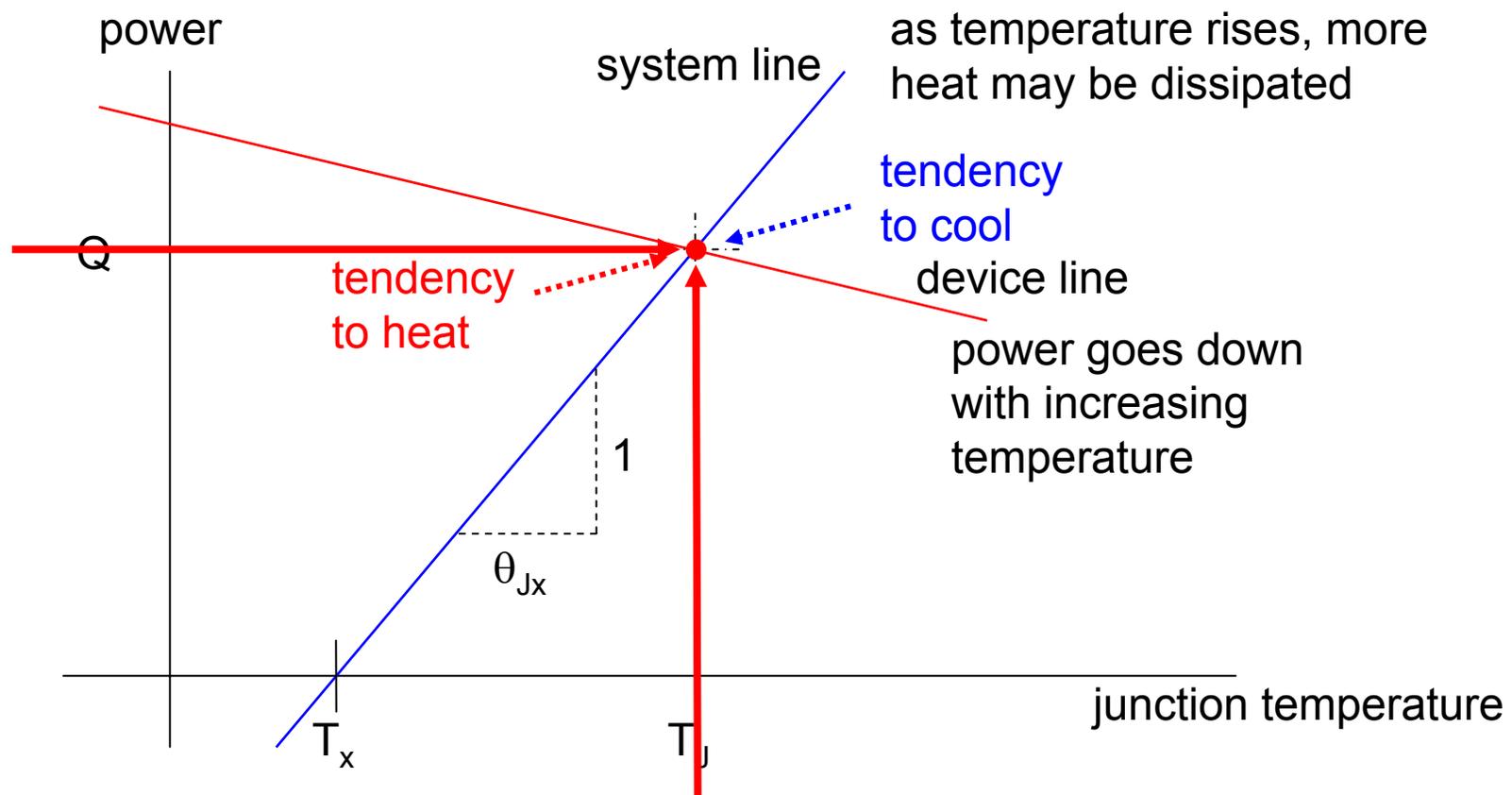
$$\frac{dQ}{dT} = \frac{1}{\theta_{Jx}}$$

sensitivity (slope) of power with
respect to temperature

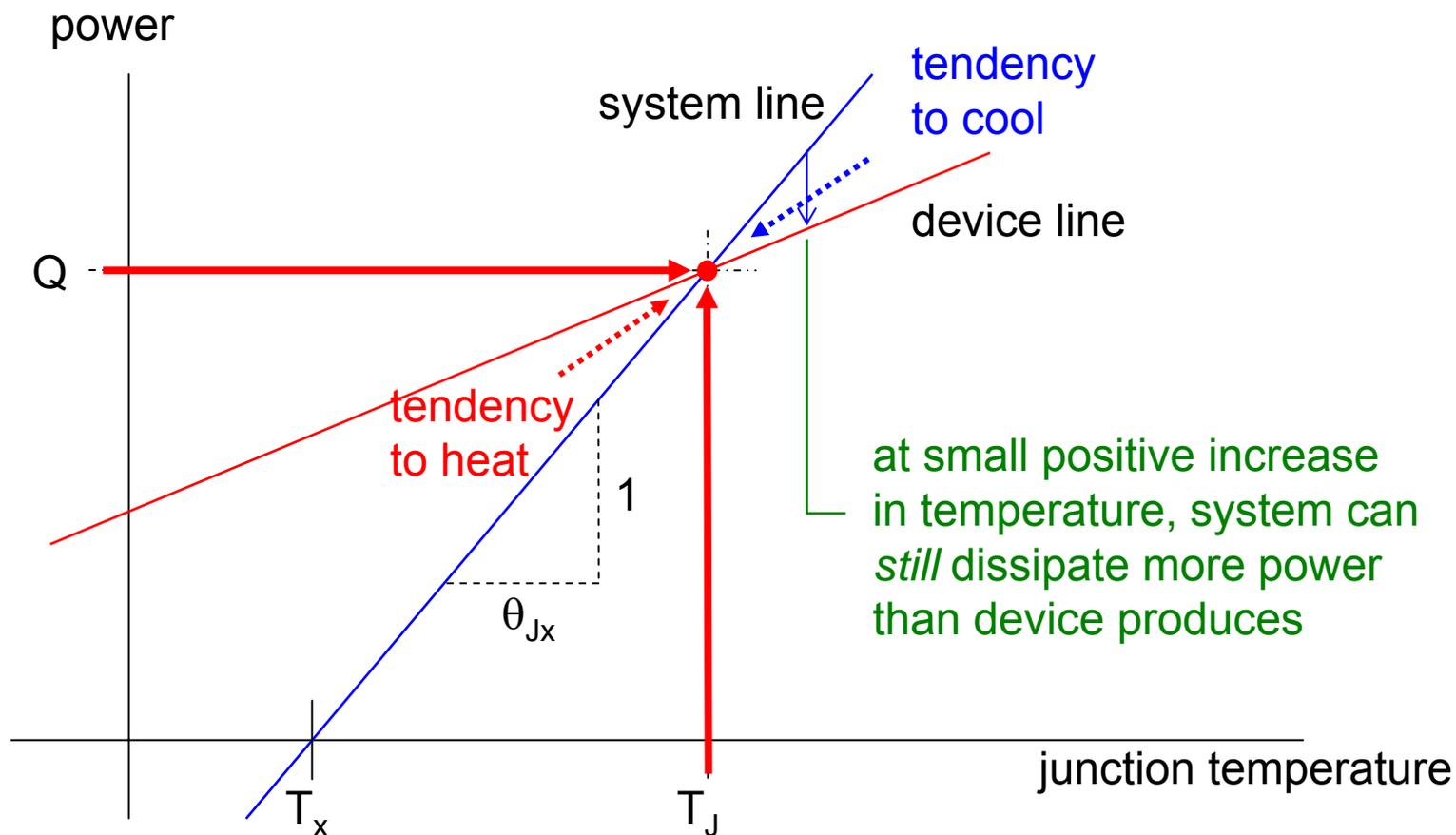
Operating point of thermal system with temperature-independent power



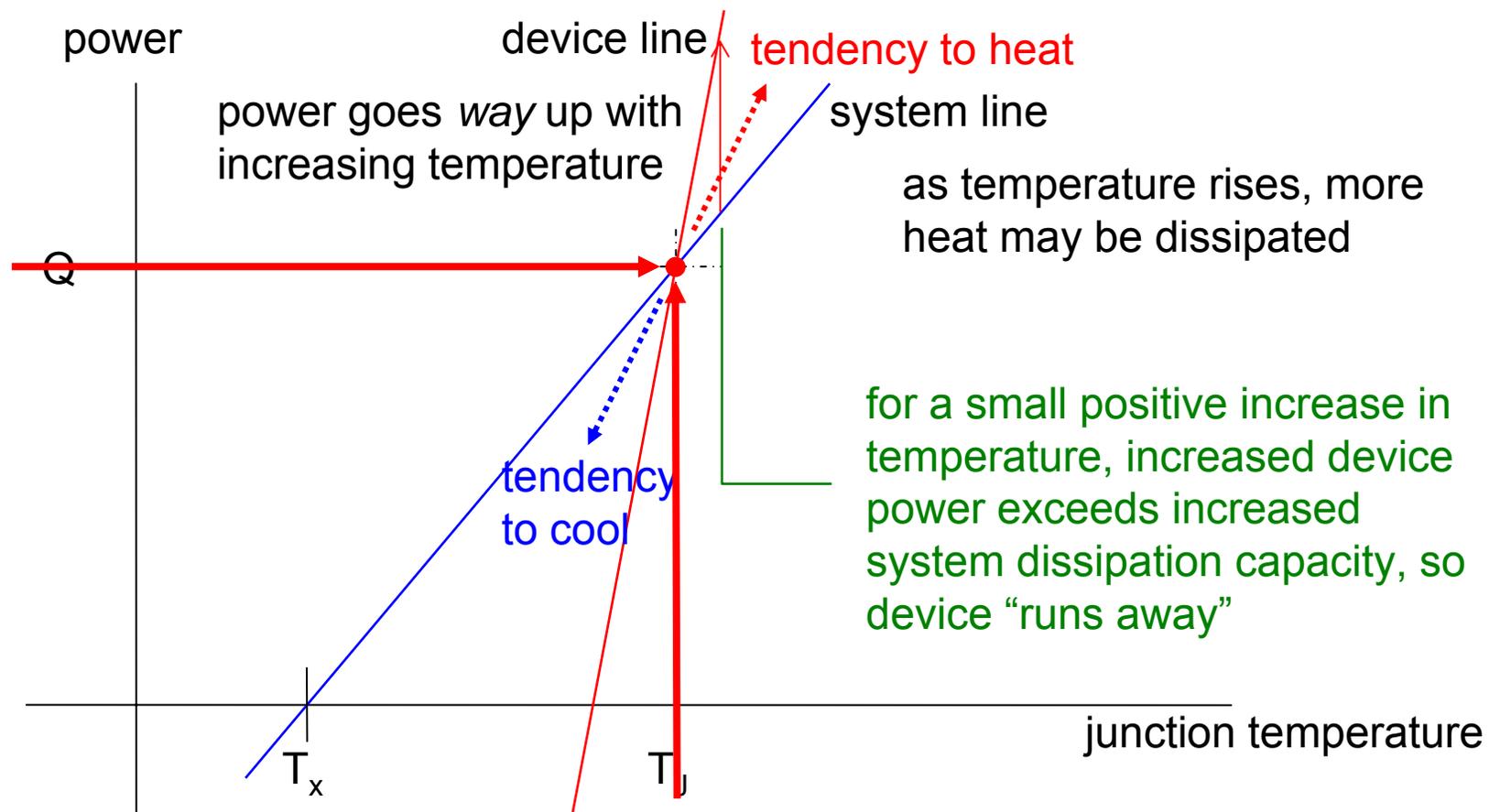
Operating point of thermal system where power *decreases* with temperature



Operating point of thermal system where power increases with temperature, slopes favorable

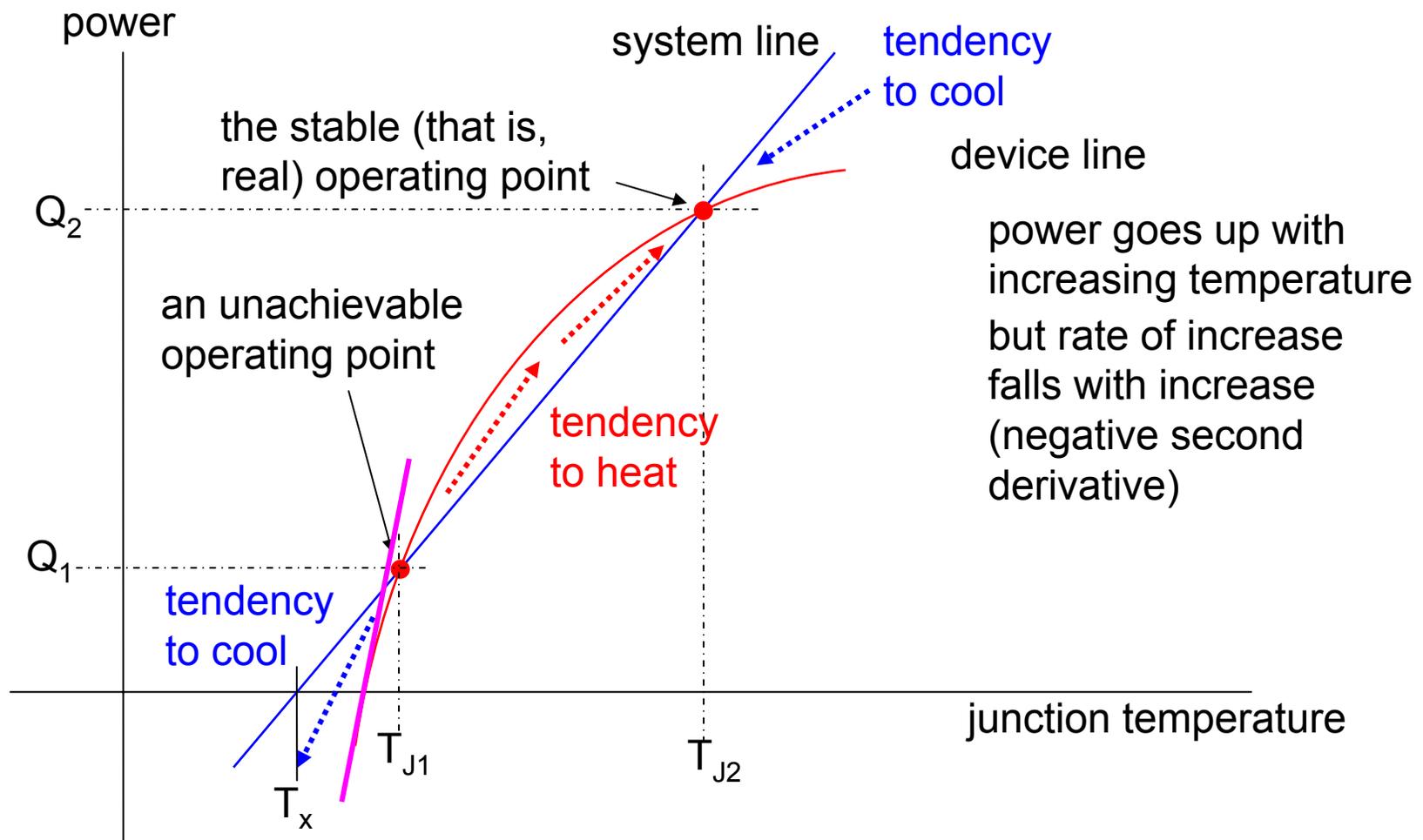


Operating point of thermal system where power increases with temperature, slopes unfavorable

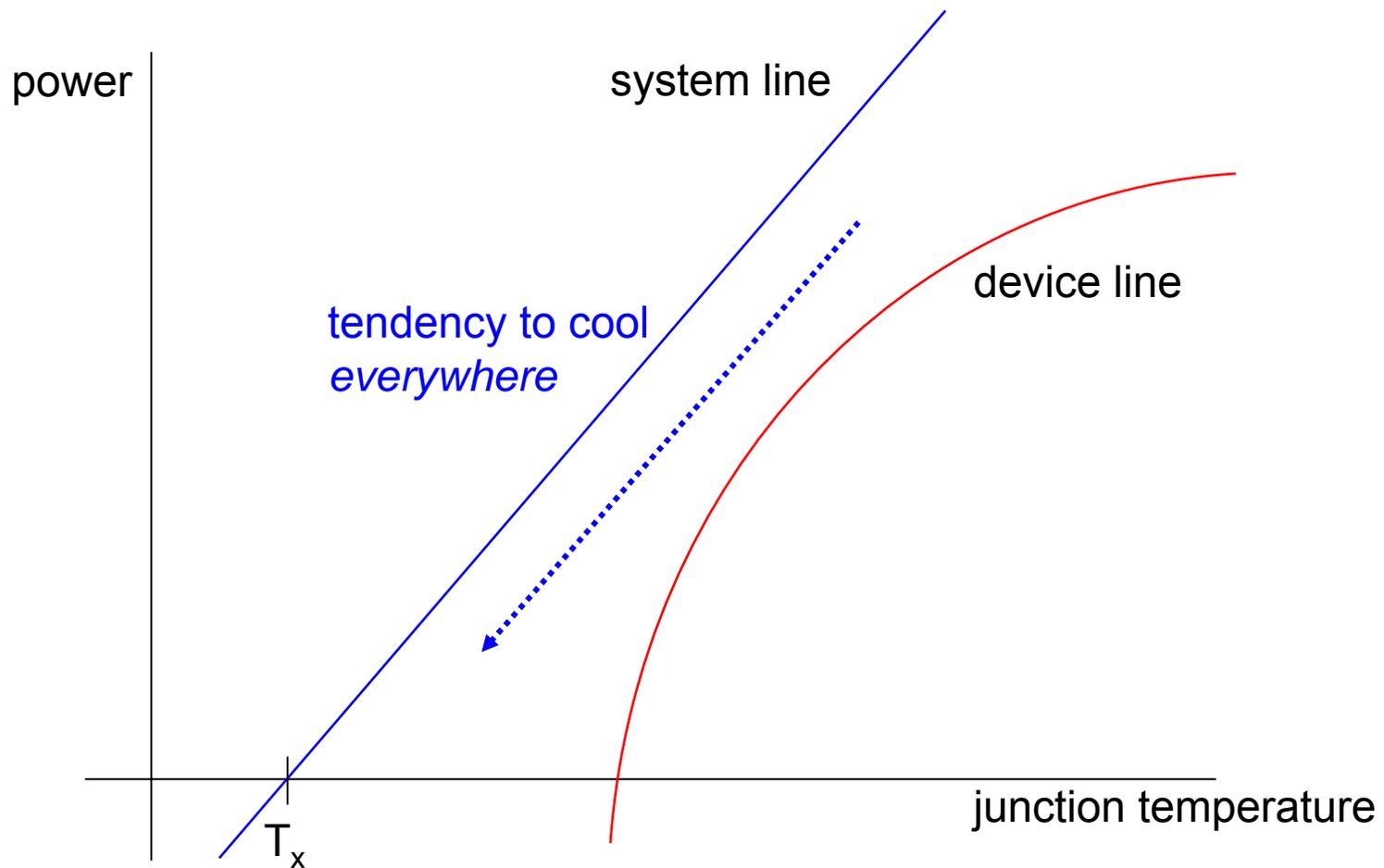




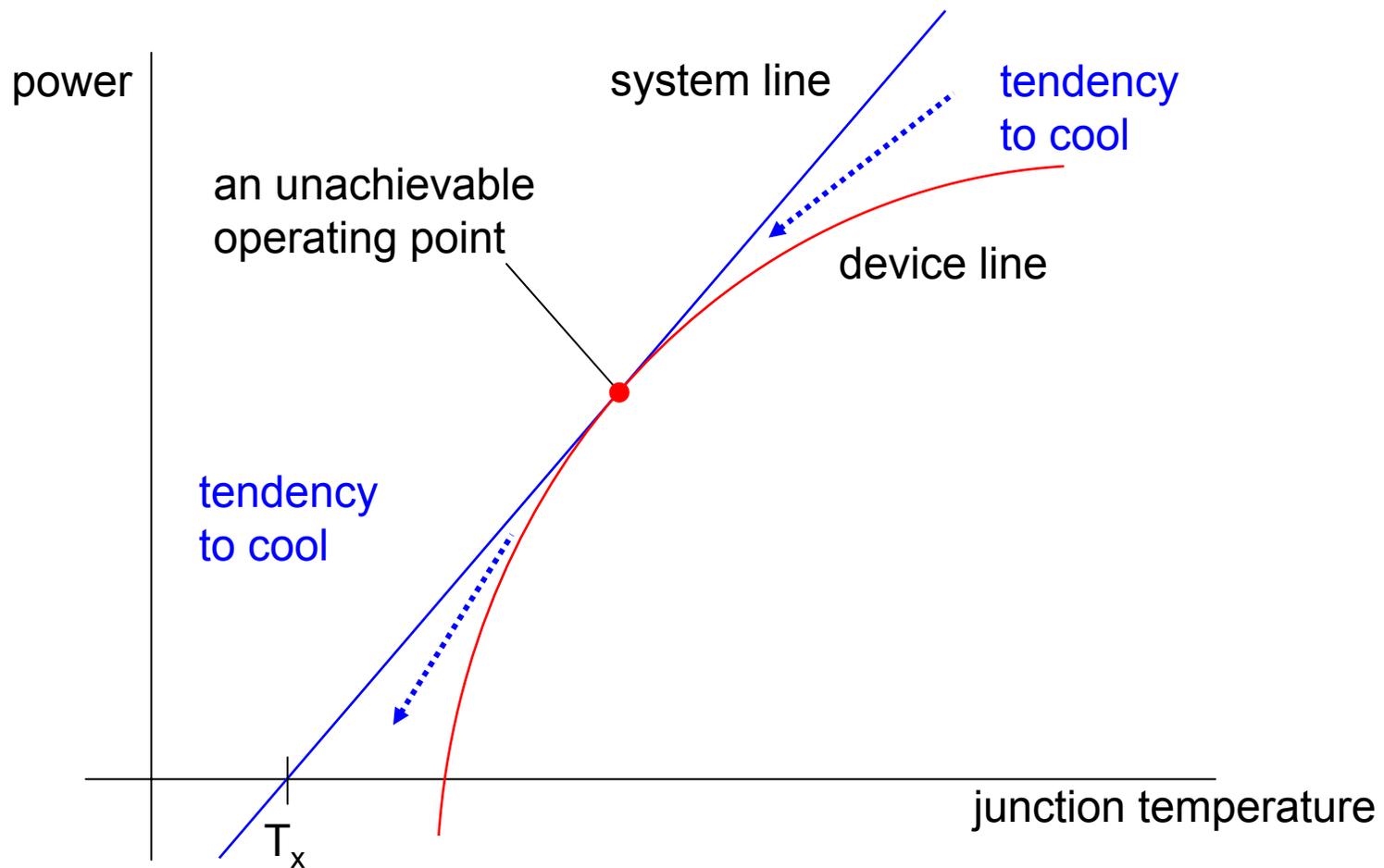
Operating points of thermal system when device line has negative second derivative



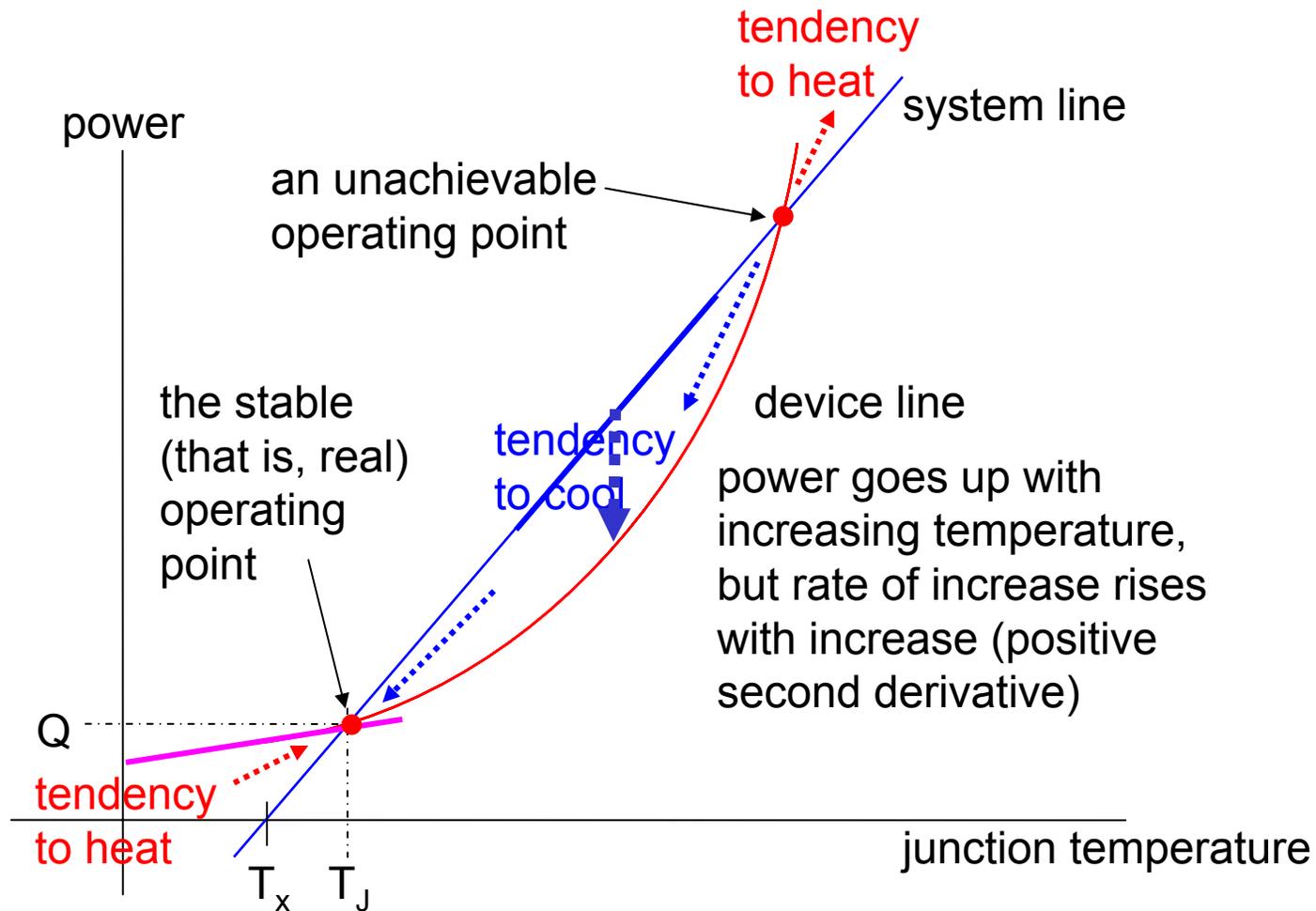
System with no operating point, negative second derivative, cannot be powered up



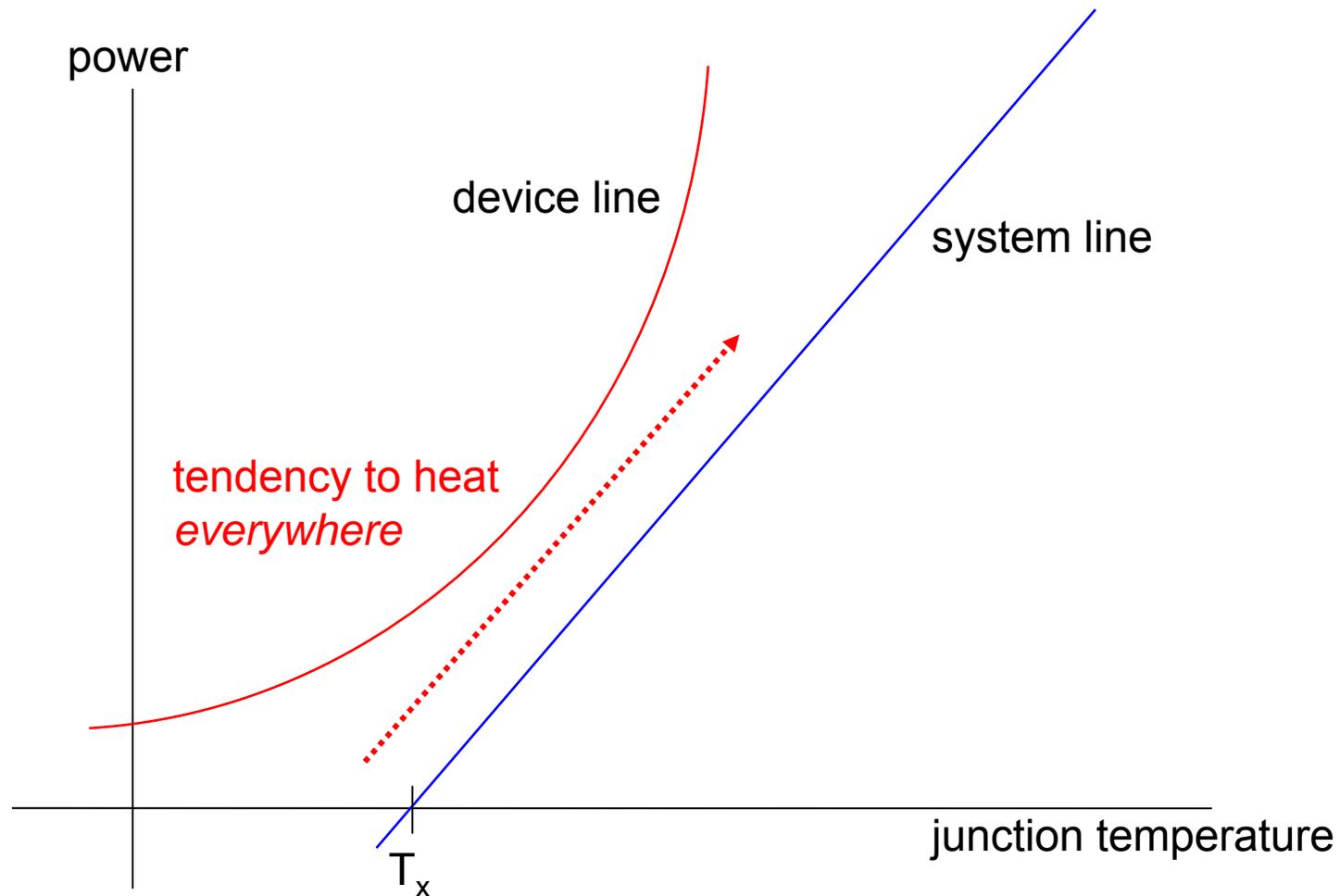
Device with negative second derivative, system has unrealizable operating point,



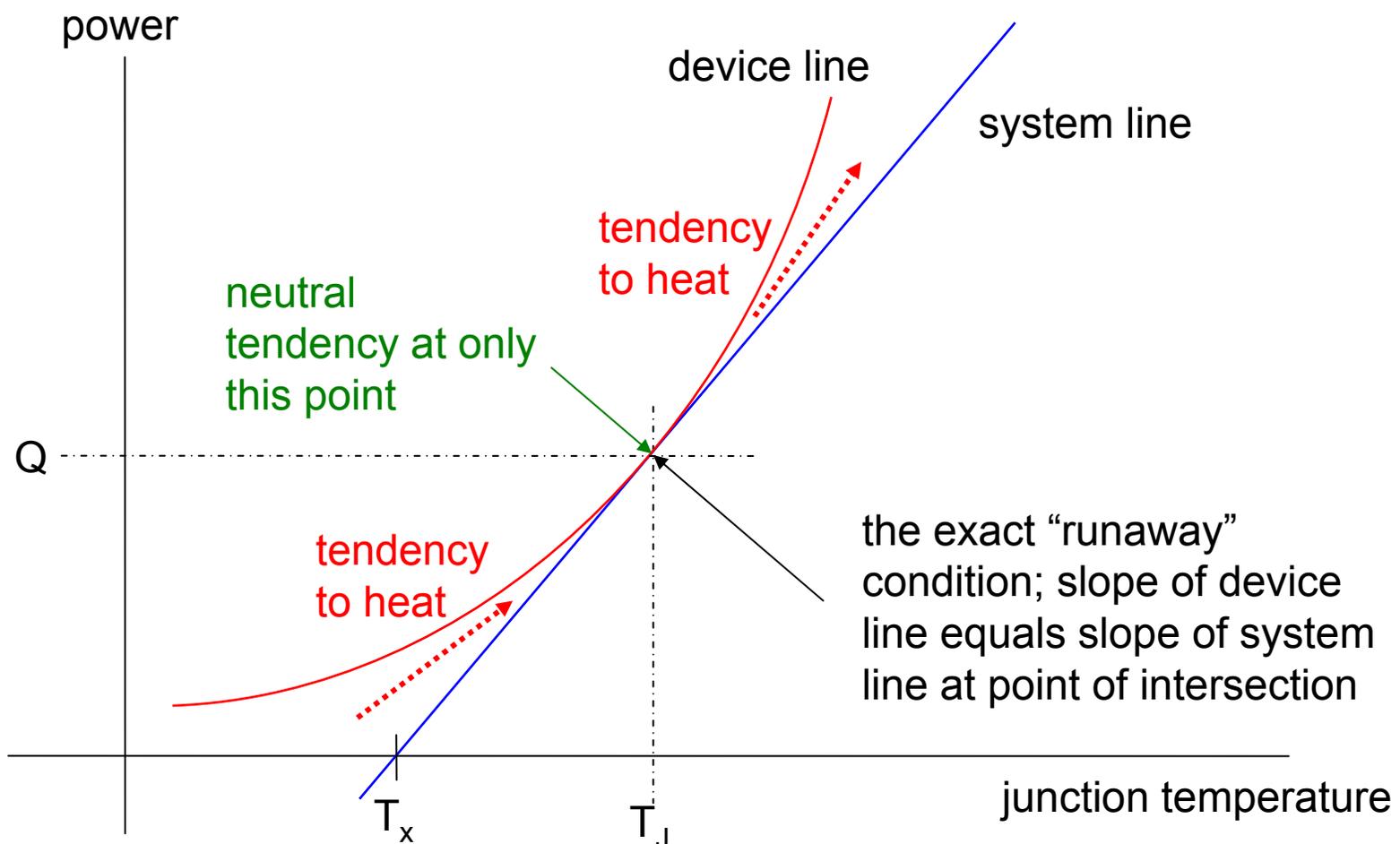
Operating points of thermal system when device line has *positive* second derivative



System with NO operating point, overheats as soon as powered up

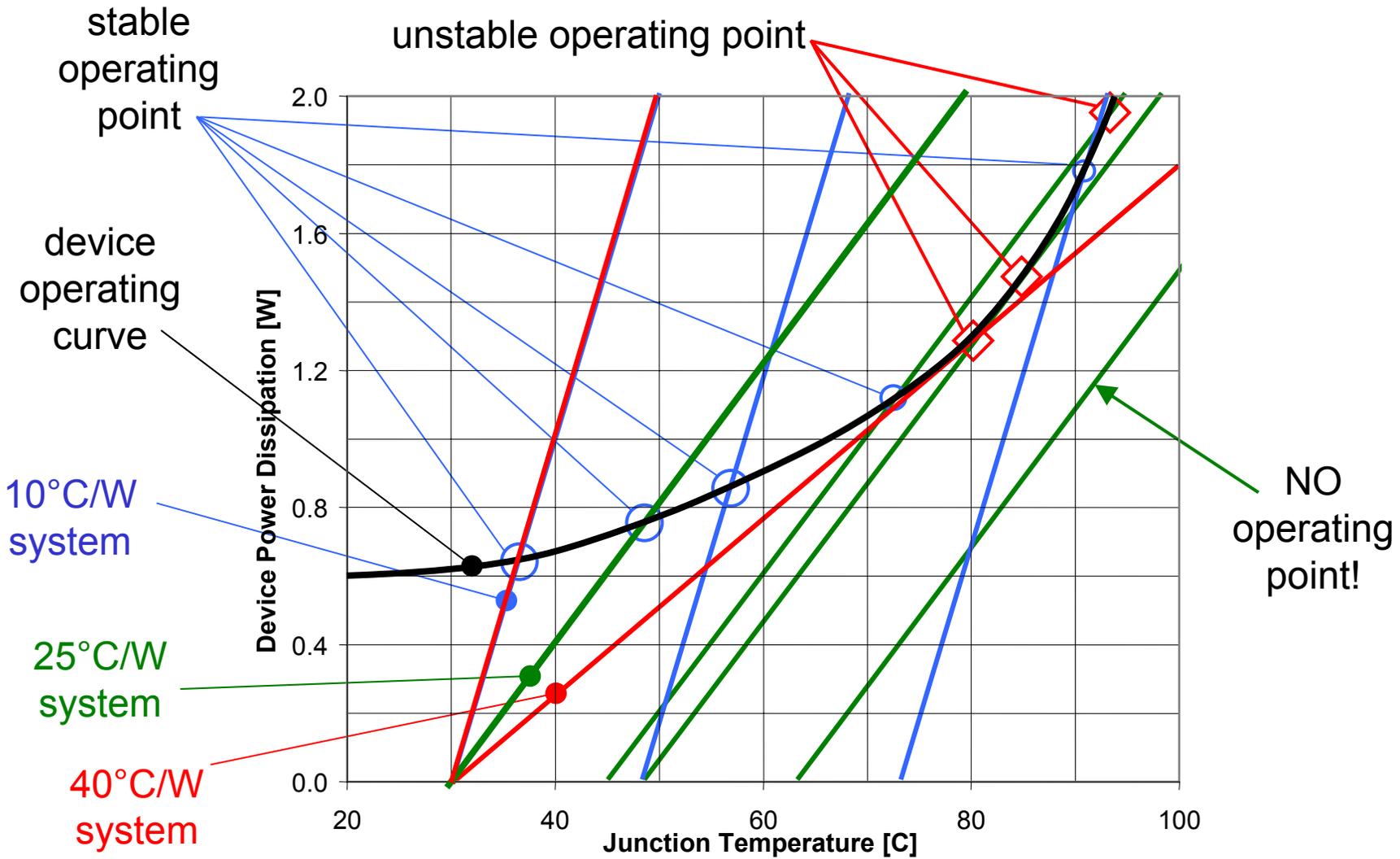


System with exactly one “runaway” operating point, device has positive second derivative



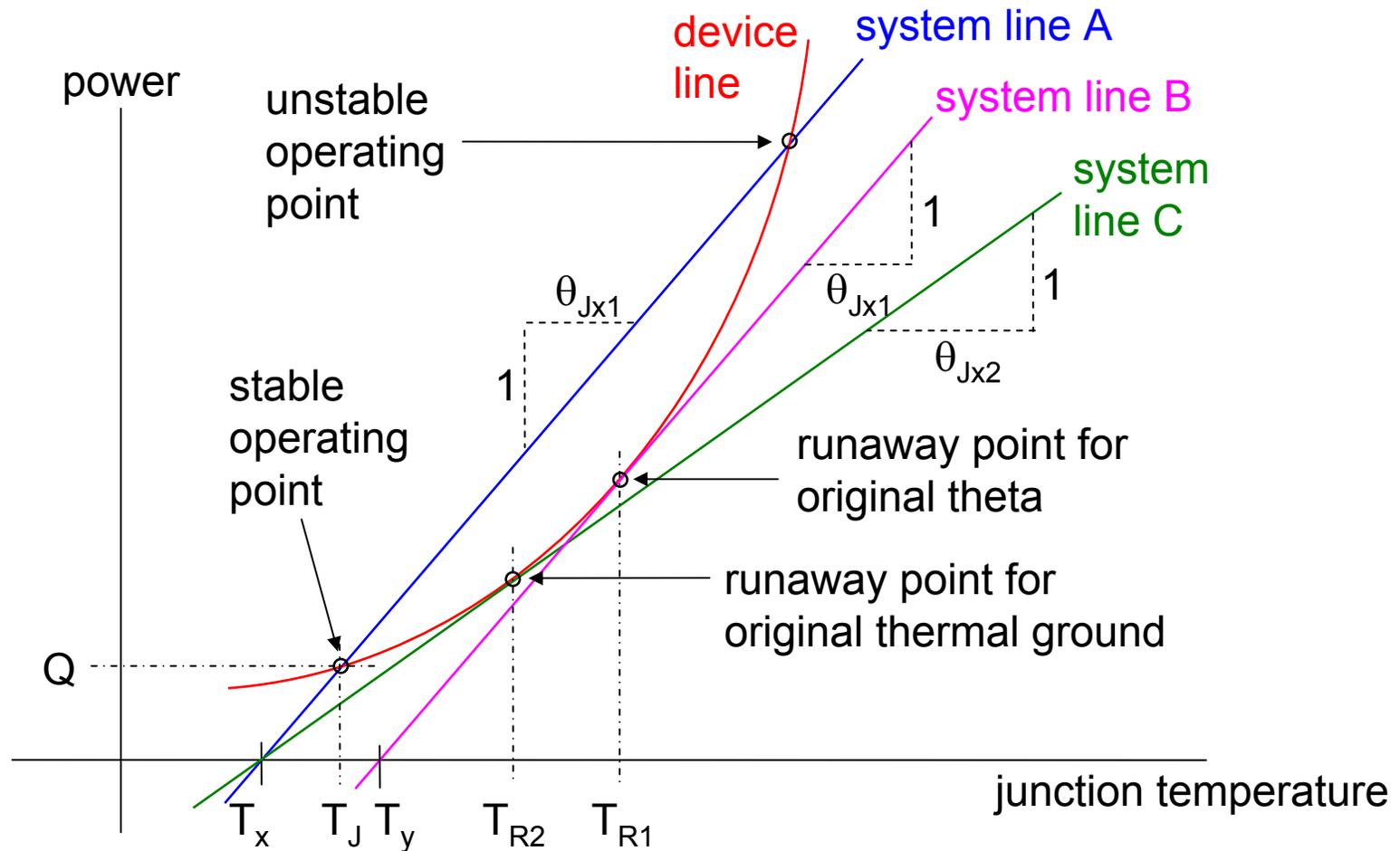


Let's see how it works





Generic power law device and generic linear cooling system



Don't get confused by the terms!

device power

$$Q = V \cdot I$$

a mathematical
“power law”

$$y = a^x$$

an “exponential”
power law (base is e)

$$y = e^x$$

Definition of power law device

rule of thumb for leakage;
2x increase for every 10°C

$$I = I_0 2^{\frac{T}{10}}$$

$$I = I_0 e^{(\ln 2) \frac{T}{10}} = I_0 e^{\left(\frac{10}{\ln 2}\right) \frac{T}{10}}$$

$$I = I_0 e^{\lambda T}$$

defining: $\lambda = \frac{T_1 - T_2}{\ln\left(\frac{I_1}{I_2}\right)}$

for constant voltage, power does
the same

$$Q = V_R I_0 e^{\lambda T} = Q_0 e^{\lambda T}$$

1st and 2nd derivatives

$$\frac{dQ}{dT} = \frac{Q_0}{\lambda} e^{\lambda T} \quad \frac{d^2Q}{dT^2} = \frac{Q_0}{\lambda^2} e^{\lambda T}$$

both always positive



The mathematical essence

System line

$$Q = \frac{T - T_x}{\theta_{Jx}}$$

Power law device line

$$Q = Q_o e^{\frac{T}{\lambda}}$$

Non-dimensionalizing

$$z = \frac{T - T_x}{\lambda} \quad \text{temperature}$$

$$q = \left(\frac{1}{Q_o} e^{\frac{-T_x}{\lambda}} \right) Q \quad \text{power}$$

Leads to:

(system)

$$q = kz$$

where:

$$k = \frac{\lambda}{\theta_{Jx} Q_o} e^{\frac{-T_x}{\lambda}}$$

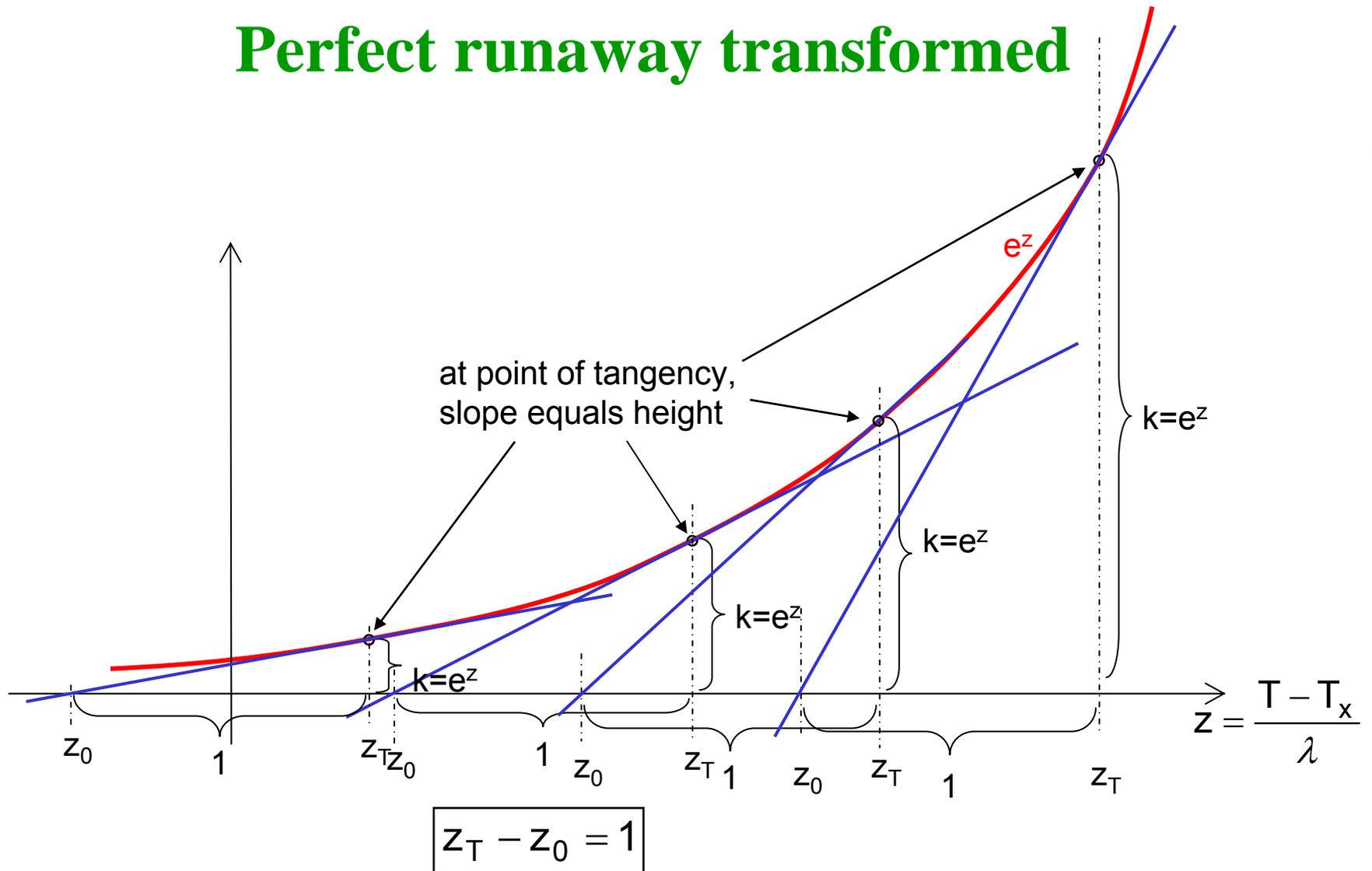
(power law device)

$$q = e^z$$

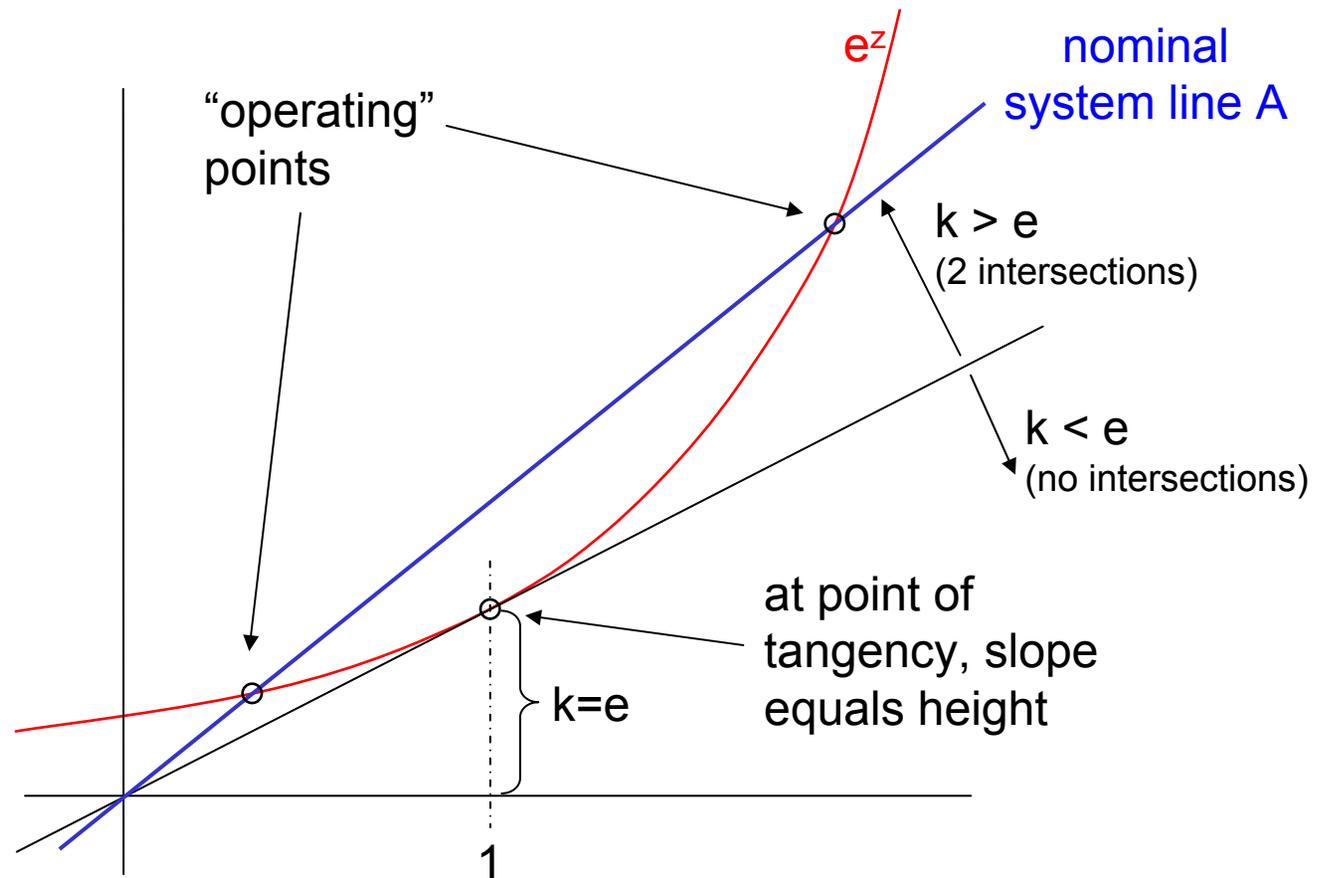
Eliminating q:

$$kz = e^z$$

Perfect runaway transformed

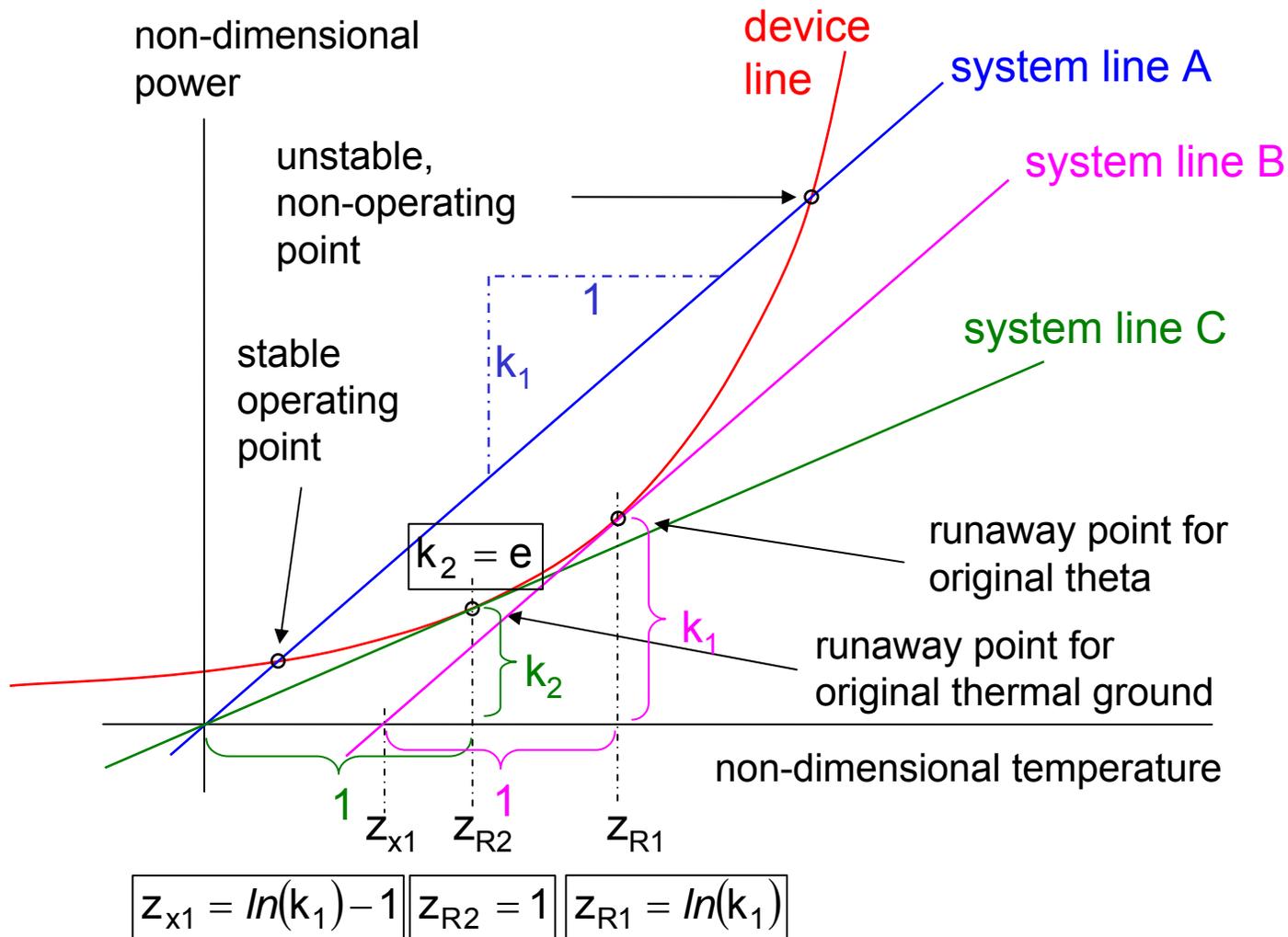


Transforming the nominal system





Everything transformed



“Perfect runaway” results in original terms

runaway temperature based on
original slope

$$T_{R1} = \lambda \ln \left(\frac{\lambda}{\theta_{Jx1} Q_o} \right)$$

max ambient that goes
with it

$$T_{x1} = \lambda \ln \left(\frac{\lambda}{\theta_{Jx1} Q_o} \right) - \lambda$$

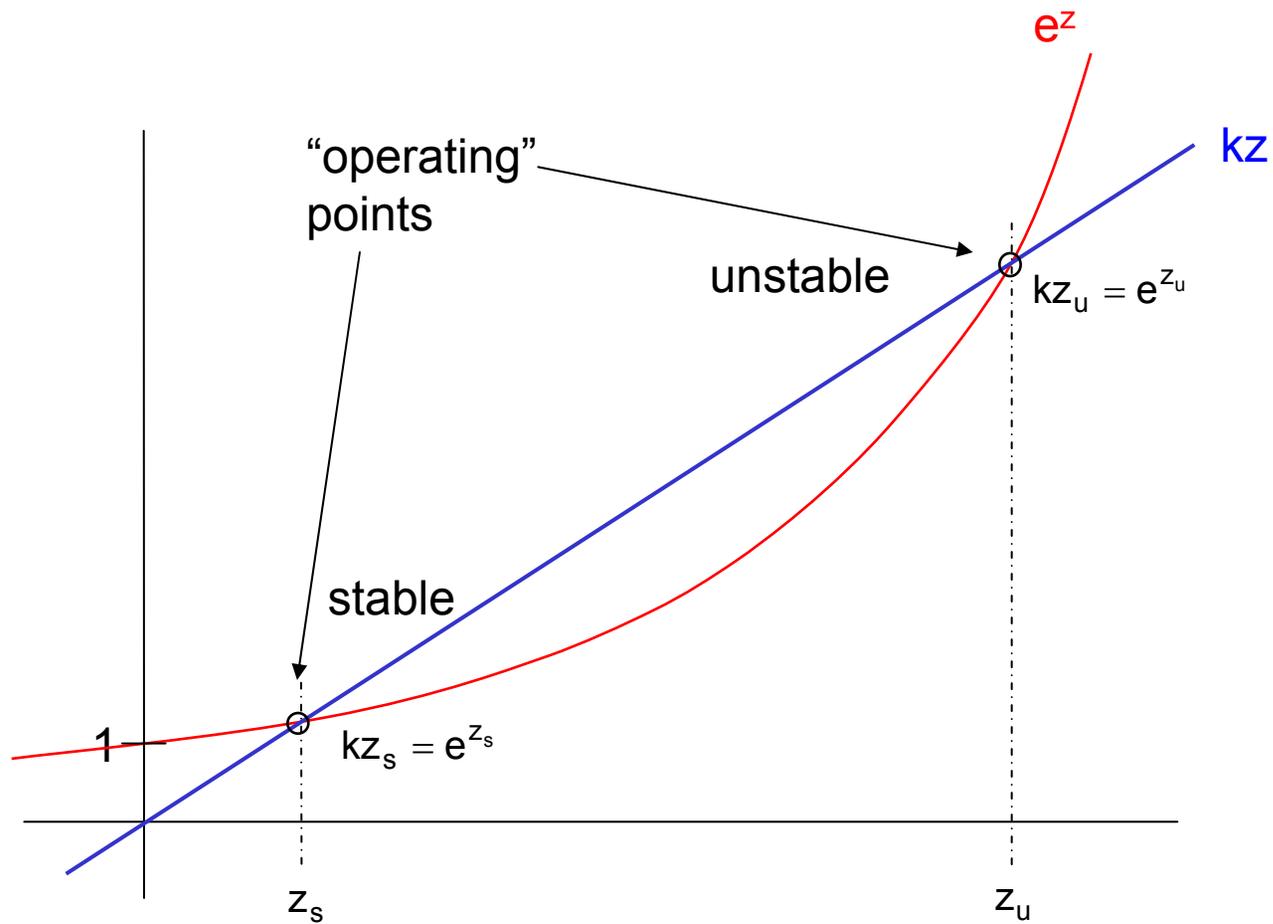
runaway temperature based on
original ambient

$$T_{R2} = T_x + \lambda$$

system resistance that
goes with it

$$\theta_{Jx2} = \frac{\lambda}{Q_o} e^{-\left(\frac{T_x}{\lambda} + 1\right)}$$

The “operating” points



Newton's method for the intersections

$$z_{i+1} = z_i - \frac{-F(z_i)}{F'(z_i)}$$

$$kz = e^z$$

$$\ln kz = z$$

$$F(z) = z - \ln kz$$

$$F'(z) = 1 - \frac{1}{z}$$

$$z_{i+1} = \frac{\ln\left(\frac{k}{e} z_i\right)}{1 - \frac{1}{z_i}}$$

For k/e ranging between 1.01 and 1000, convergence is to a dozen significant digits in fewer than 10 iterations.

$$z_0 = \frac{1}{k} = \frac{1}{e \cdot \frac{k}{e}}$$

this initial guess
converges to lower,
stable point

this initial guess
converges to upper,
unstable point

$$z_0 = \ln k = 1 + \ln\left(\frac{k}{e}\right)$$

And the intersection points come from ...

find the non-dimensional intersections first, then

$$T_{\text{stable}} = T_x + \lambda \cdot Z_{\text{stable}}$$

$$T_{\text{unstable}} = T_x + \lambda \cdot Z_{\text{unstable}}$$



Real datasheet example

raw device data†

V_r [V]	12	40
T_{max} [°C]	125	125
T_{ref} [°C]	75	75
I_{tmax} [A]	8.50E-3	2.80E-2
I_{tref} [A]	5.20E-4	1.70E-3

$$I = I_0 e^{\frac{T}{\lambda}}$$

$$I_0 = I_{tmax} e^{-\frac{T_{max}}{\lambda}} = I_{tref} e^{-\frac{T_{ref}}{\lambda}}$$

the device power curve parameters

	@12V	@40V
λ [°C]	17.9	17.8
Q_0 [W]	9.4E-5	1.02E-3

$$\lambda = \frac{T_{max} - T_{ref}}{\ln\left(\frac{I_{max}}{I_{ref}}\right)}$$

rule of thumb gave us: $= \frac{10}{\ln(2)} = 14.4$

$$Q_0 = V_R I_0$$

† MBR140T3



Runaway analysis in nominal system

computed results

raw device data†

V_r [V]	12	40
T_{max} [°C]	125	125
T_{ref} [°C]	75	75
I_{tmax} [A]	8.50E-3	2.80E-2
I_{tref} [A]	5.20E-4	1.70E-3

	@12V	@40V	@40V
λ [°C]	17.9	17.8	
Q_o [W]	9.4E-5	1.02E-3	

k/e (compare to unity)		10.6	0.97	1.609
given theta	T_x max [°C]	117.2	74.4	83.5
	T_{R1} [°C]	135.1	92.2	101.3
given ambient	θ_{Jx2} max [°C/W]	1055	96.6	
	T_{R2} [°C]	92.9	92.8	

$$\frac{k}{e} = \frac{\lambda}{\theta_{Jx} Q_o} e^{-\frac{T_x}{\lambda} - 1}$$

$$T_x = 75$$

$$\theta_{Jx1} = 100$$

$$\theta_{Jx1} = 60$$

These translate into:

a stable operating point at 80.6°C (and 0.09 W),

an unstable point at 116.3°C 0.69 W

$z = 0.312$

$z = 2.315$

† MBR140T3

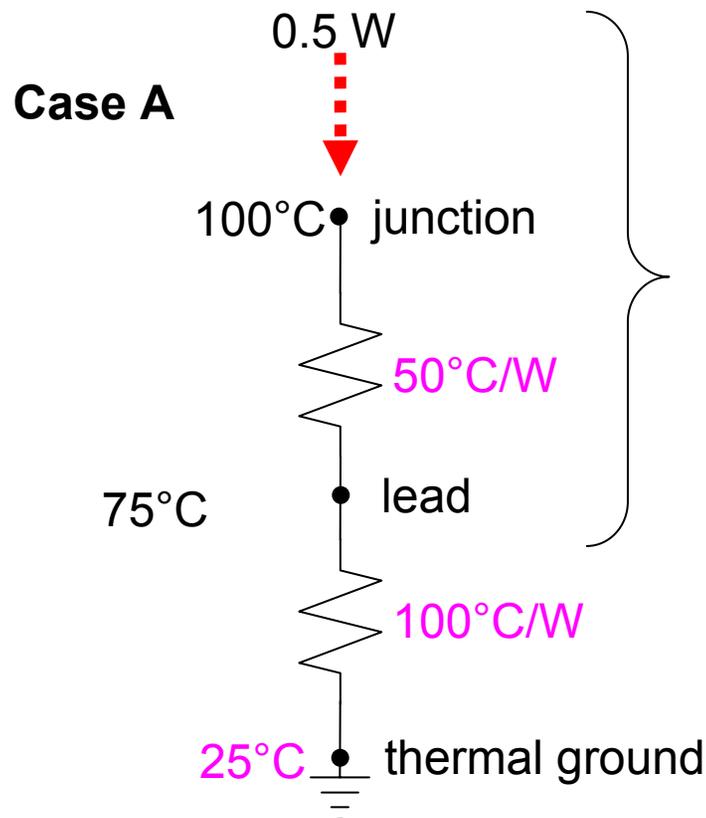


How about the real thermal system?

- Is ambient really ambient?
- Is theta-JA what you think it is?

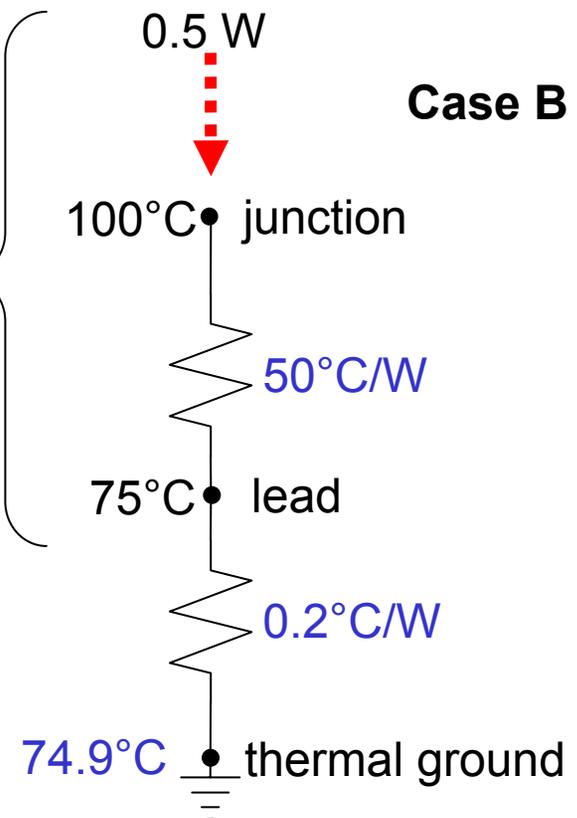


A paradox



thermal runaway,
based on $\theta_{Jx} = 150^\circ\text{C/W}$,
calculated to be at **125°C**

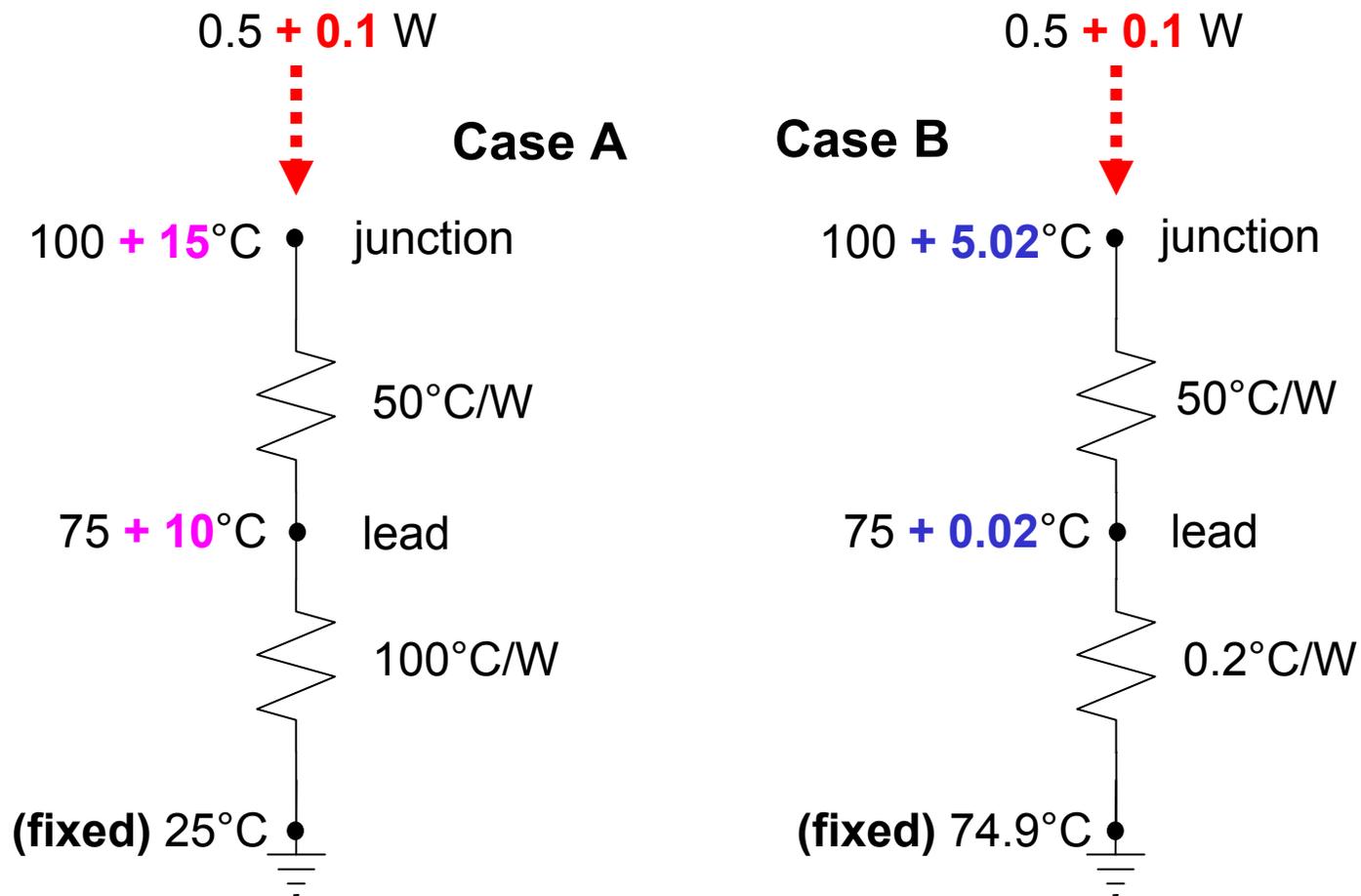
identical



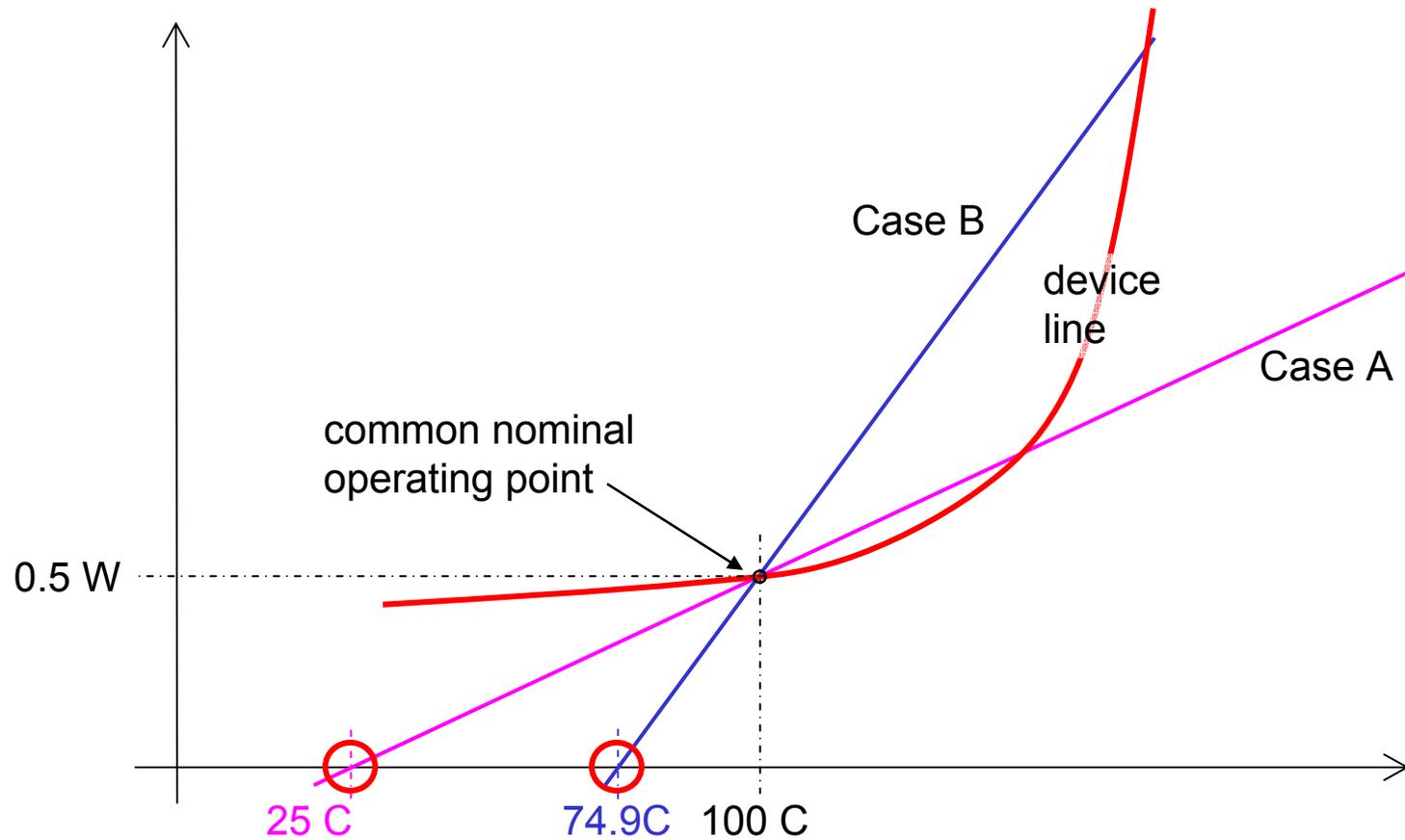
thermal runaway,
based on $\theta_{Jx} = 50.2^\circ\text{C/W}$,
calculated to be at **150°C**

Paradox lost

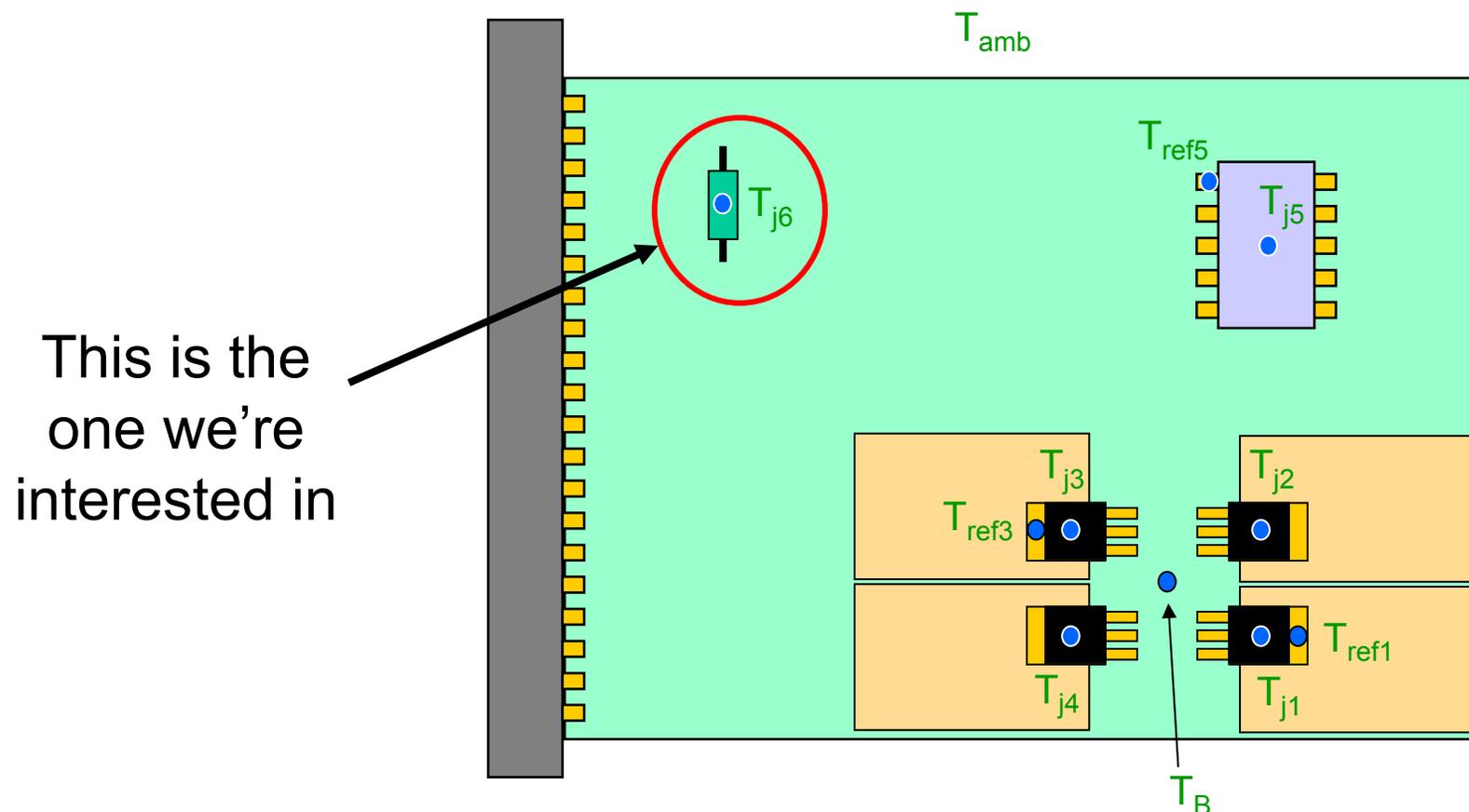
raise the power by 0.1 W and see what happens



Illustrating the paradox



Consider the following 6-component example of a complete system, using linear superposition to describe the thermal behavior



This is the one we're interested in

Linear superposition math

$$T_j = \theta \cdot Q + T_a$$

matrix product

temperature (vector)

theta (matrix)

power (vector)

ambient (scalar)

The diagram illustrates the linear superposition equation $T_j = \theta \cdot Q + T_a$. The terms are annotated as follows: T_j is the temperature (vector); θ is the thermal resistance (matrix); Q is the power (vector); and T_a is the ambient temperature (scalar). A bracket above the $\theta \cdot Q$ term is labeled 'matrix product'. Red arrows point from the text labels to their respective variables in the equation.

Putting illustrative numbers on the problem:

theta array

J1	75	65	55	60	22	10
J2	65	71	60	55	25	11
J3	55	60	65	61	21	15
J4	60	55	61	73	18	11
J5	22	25	21	18	125	14
J6	10	11	15	11	14	180
R1	73	65	55	59	22	10
R3	55	60	63	61	21	15
R5	20	24	14	19	95	15
B	65	63	62	63	21	12

power
vector

Q_{j1}	0.5
Q_{j2}	0.5
Q_{j3}	0.5
Q_{j4}	0.5
Q_{j5}	0.2
Q_{j6}	0.02

Observe the relative contributions

the other devices ...

$$= (10 \times 0.5) + (11 \times 0.5) + (15 \times 0.5) + (11 \times 0.5) + (14 \times 0.2) + (180 \times 0.02)$$

the device itself ...

$$= 5.0 + 5.5 + 7.5 + 5.5 + 2.8 + 3.6 + 25 + 25$$

(ambient)

$$= 26.3 +$$

$$3.6 +$$

$$25$$

(ambient)

Symbolically, for just T_{j6} , we'd write this:

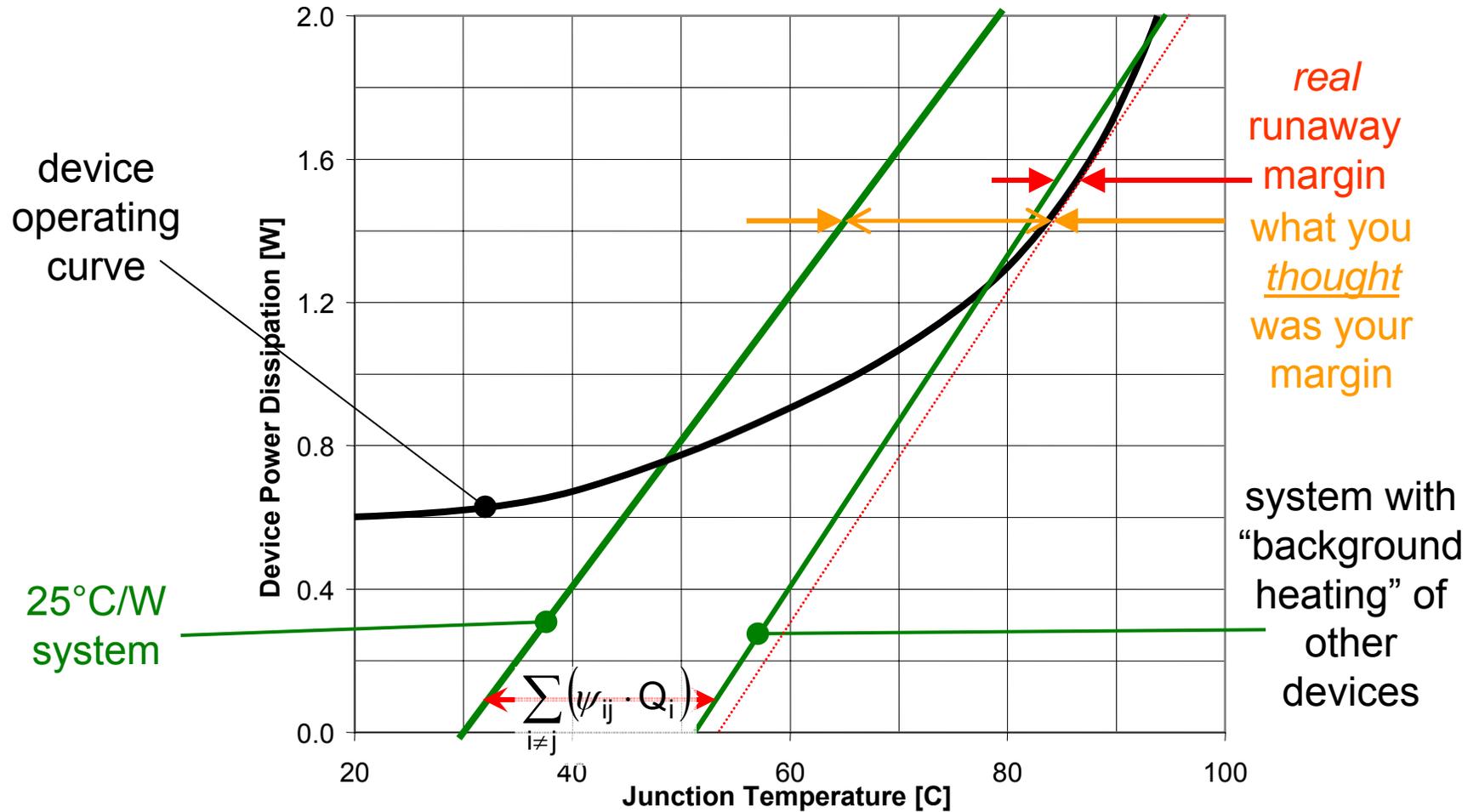
$$T_{j6} = \sum_{i=1}^5 (\psi_{i6} \cdot Q_i) + \theta_{j6a} \cdot Q_6 + T_a$$

“system” slope
 effective ambient
 +
 temperature of device #6
 power of other devices
 power of device #6
 ambient (scalar)

“interaction” terms from theta matrix (off-diagonal elements)
 device #6 “self heating” term from theta matrix



Graphically, it looks like this





How does *effective ambient* relate to board temperature?

“system” slope for isolated device

if any of *these* are non-zero, T'_a will be higher than T_a

$$\begin{aligned}
 T_{j6} &= \theta_{j6a} \cdot Q_6 + \sum_{i=1}^5 (\Psi_{i6} \cdot Q_i) + T_a \\
 &= (\theta_{j6B} + \theta_{B6a}) \cdot Q_6 + \underbrace{\sum_{i=1}^5 (\Psi_{i6} \cdot Q_i) + T_a}_{\text{effective ambient } T'_a} \\
 &= \theta_{j6B} \cdot Q_6 + \theta_{B6a} \cdot Q_6 + \\
 &= \Delta T_{j6B} + \Delta T_{B6a} +
 \end{aligned}$$

temperature rise, board to J6
temperature rise, ambient to board

when Q_6 is zero, T_{j6} will be zero. Note that these will be zero if the board is non-zero



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