COMPLEXITY REDUCTION OF PARTIAL UPDATE OVERSAMPLED SUBBAND ADAPTIVE ALGORITHMS BY SELECTIVE PRUNING OF POLYPHASE COMPONENTS

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ABSTRACT

Partial update algorithms such as sequential LMS have been proposed to reduce computation cost. To further decrease the computation cost for low-resource real-time echo cancellation by oversampled subband adaptive filters (OS-SAFs), we propose a new hybrid partial update algorithm. This algorithm, based on a polyphase perspective, selectively prunes (zeros out) polyphase components of the adaptive filter. When employed on OS-SAFs, the proposed algorithm achieves significant complexity reduction. This algorithm is applied to sequential versions of LMS and Pseudo-Affine Projection algorithms employed to adapt OS-SAFs. Presented performance evaluations prove that considerable computation cost reduction (typically more than four times) is feasible with practically insignificant degradations in the steady-state performance of the OS-SAF system. Moreover, the algorithm does not affect initial convergence properties.

1. INTRODUCTION

In real-time adaptive applications such as acoustic echo cancellation, there is often a need to adapt long filters. Partial update algorithms such as sequential normalized LMS (S-NLMS) [1] have been employed to reduce the computation cost. Computation reduction is also available from frequency-domain systems based on oversampled subband adaptive filters (OS-SAF) due to their fast convergence, low-delay, and low-distortion characteristics [2].

Fig. 1 depicts a typical OS-SAF system with each subband adaptive filter consisting of an FIR filter. The employed filterbank in this research is a weighted overlap-add (WOLA) filterbank used in many low-power tasks [3]. The WOLA filterbank has many associated advantages and of particular note is its capability for high over-sampling factors permitting the use of a relatively short and low-delay prototype filter with a long frequency transition region [3].

However, oversampling results in a coloration of the subband signals tending to hinder the NLMS convergence. To mitigate this effect, we have already proposed a method of whitening by decimation (WBD) of the OS-SAF subband inputs [2]. The method employs decimated versions (by factor D) of the bandlimited subband signals ($x_k(m)$, $y_k(m)$ for subband k) for adaptation as depicted in Fig. 2. The wider bandwidth of the decimated signals leads to faster convergence of the subband adaptive filter $w'_k(\cdot)$. This filter is then interpolated to obtain
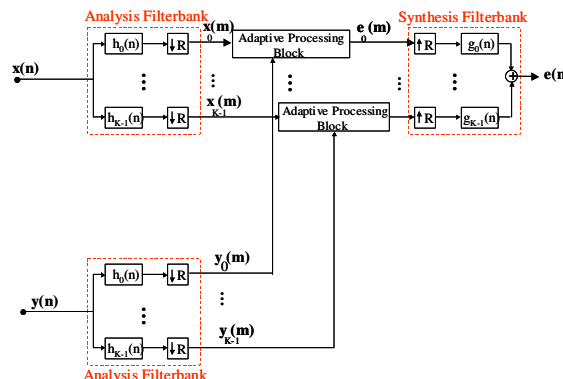

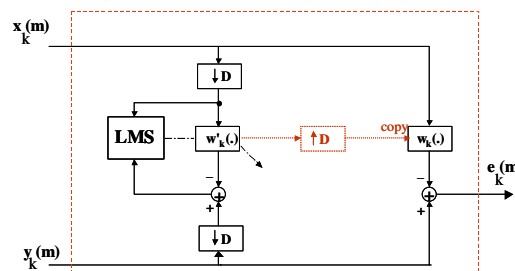Figure 1: Block diagram of the OS-SAF system

Figure 2: Whitening by decimation of the subband signals.

the adaptive filter $w_k(\cdot)$ in the main branch. Obviously decimation leads to aliasing (only in the side branch) and performance limitation as D increases [2,4]. To limit the aliasing and its adverse effects on asymptotic performance of the OS-SAF system, only small values of D ($D \le OS/2$, where OS denotes the filterbank oversampling ratio) are allowed. As a result, only limited complexity reduction is achievable by this method and alternative solutions should be investigated.

Recent analysis of S-NLMS (as well as other partial update methods) in [5] has shown its equivalence to a perfect reconstruction (PR) delay chain filterbank. Basically, S-NLMS decomposes the adaptive filter into its polyphase components to be adapted separately and at different time instances. This analysis was then extended to the WBD technique described in [2,4] to show that WBD is equivalent to S-NLMS whilst ignoring (zeroing out) all but one of the polyphase components of the adaptive filter [5]. Similar analysis was presented for the Sequential Gauss-Seidel Pseudo-Affine Projection (SGS-PAP) algorithm in [6].

In this paper, a new adaptation algorithm is proposed for OS-SAF systems to further reduce the computation cost. This method is a hybrid of WBD and partial update algorithms, with a decimation rate not limited by the over-sampling factor. The key is to selectively ignore some (rather than all but one, as in WBD) polyphase components. Just as in WBD, the resulting aliasing can be shown to be negligible when using high over-sampling factors. This pruning of specific polyphase components gives the algorithm an advantage in computational complexity versus S-NLMS and SGS-PAP in both filtering and adaptation. For brevity, presented algorithm development and analysis is limited to the S-NLMS but the method could be similarly combined with other partial update algorithms. As an example, evaluation results for SGS-PAP are actually presented in the paper.

The algorithm is described in Section 2, and an analysis is presented in Section 3. Simulation of the algorithm was done with an OS-SAF weighted overlap-add filterbank [3] based on a generalized DFT filterbank. Results of this simulation are presented and examined in Section 4, and conclusions are drawn in Section 5.

2. ALGORITHM DESCRIPTION

Sequential LMS is described and analyzed by statistical analysis in [1]. S-NLMS is then reformulated from a multirate processing viewpoint leading to a decomposition of the adaptive filter into polyphase components in [5]. Using terminology from both of these references, the S-NLMS adaptation equation for a filter of length L and a decimation factor of D is given by,

$$w_i(n+1) = \begin{cases} w_i(n) + \dfrac{\mu e(n) x(n-i)}{(L/D)\sigma_x^2} & \text{if } (n-i) \bmod D = 0 \\ w_i(n) & \text{otherwise} \end{cases} \quad (1)$$

$$e(n) = y(n) - \mathbf{W}^T(n)\mathbf{X}(n) \quad (2)$$

where $\mathbf{W}(n) = [w_0(n), w_1(n),...,w_{L-1}(n)]^T$ is the tap weight vector of the adaptive filter, $\mathbf{X}(n) = [x(n), x(n-1),...,x(n-L+1)]^T$ is the reference input vector, $y(n)$ is the primary input signal, $e(n)$ is the error signal and σ_x^2 is the variance of the reference signal.

The proposed algorithm contains a modification to the S-NLMS using another variable, called the imaging factor (described in Section 3), denoted by I. The new adaptation equation is given by,

$$w_i(n+1) = \begin{cases} 0 & \text{if } i \bmod I \neq 0 \\ w_i(n) + \dfrac{I\mu e(n) x(n-i)}{(L/D)\sigma_x^2} & \text{if } (n-i) \bmod D = 0 \\ w_i(n) & \text{otherwise} \end{cases} \quad (3)$$

For $I=1$, (3) reduces to S-NLMS, and for $I=D$ it reduces to WBD. The L/D factor in the denominators of (1) and (3) is because the effective length of each polyphase component is now L/D rather than L. It has also been observed that using a step-size of $D\mu$ in S-NLMS would lead to identical convergence performance (for stationary plants) across various D values [5]. The scaling by I in (3) appears due to decimation by I in the algorithm. Our evaluations (reported in Section 4) demonstrate that step-size scaling by $D.I$ in (3) leads to very similar convergence properties for different D and I values.

3. ANALYSIS

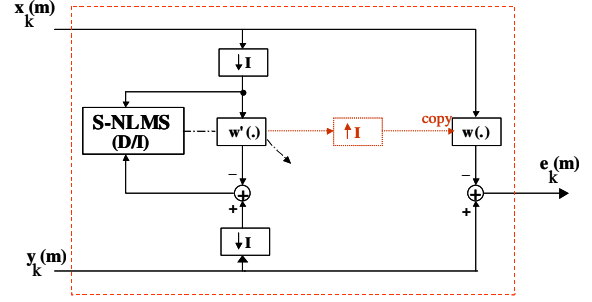There are two ways of conceptualizing the proposed

Figure 3: The proposed algorithm interpreted as WBD with S-NLMS adaptation.

algorithm. Let it be first considered as WBD with the NLMS algorithm replaced by S-NLMS. The block diagram in Fig. 3 presents this interpretation. As shown, the input signals are decimated by I in the side branch, then undergoing another decimation of D/I as part of the inherent decimation in the S-NLMS. This brings the total decimation performed on the reference signal to D. The adaptive filter in the side branch is then interpolated by a factor of I to obtain the main-branch adaptive filter. This creates filter images that are filtered out by the main branch signal $x(n)$. We call I the imaging factor to emphasize this, and to discriminate it from the decimation factor in the S-NLMS.

Using the interpretation presented by Fig. 3 one can readily determine the computational advantage of the proposed algorithm over S-NLMS. When filtering in the main branch, multiplications by zero-coefficients are removed, resulting in a computation reduction by I for filtering. Furthermore, the reductions in adaptation rate and the side branch filter length by a factor of I result in a total computational saving of I^2 for adaptation. Also, the error signal need only be taken from the main branch when its value is required during adaptation.

Fig. 3 also allows for simple comparison of the proposed algorithm with WBD. Since the imaging factor I suffers from the same limitation as the decimation factor in WBD, there is no computational advantage during filtering. The advantage comes from using S-NLMS in place of NLMS. This exchange reduces computation of the proposed algorithm by D/I during adaptation. Moreover, it is important to note that D is not limited by over-sampling. Hence, D has more room to be increased to match the resources of the target platform compared to the decimation factor in WBD.

The second means of conceptualizing the proposed algorithm starts by viewing S-NLMS as a polyphase filterbank with perfect reconstruction delay chain analysis/synthesis filters as suggested in [5]. To describe this view, consider the simple case of $D=2$ in Fig. 4. The equivalent polyphase model is presented in Fig. 5, where the polyphase filters should be adapted jointly, as suggested in [7]. Equivalently, as Fig. 6 depicts, each of the two error signals ($e_0(n)$ and $e_1(n)$) can be employed sequentially to adapt the polyphase components for all input samples. Finally, if only even samples of $x(n)$ are used for adaptation (i.e. the thin dashed lines in Fig. 6 are disconnected), we obtain the S-NLMS system for $D=2$.

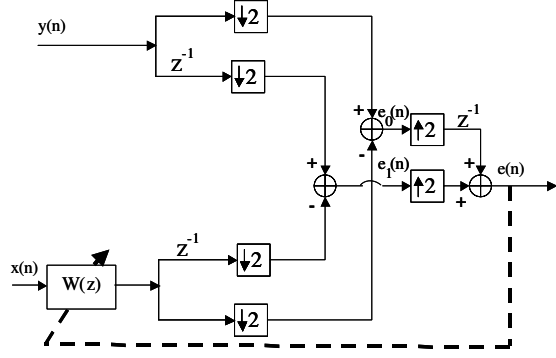In general, the D polyphase components of the adaptive filter

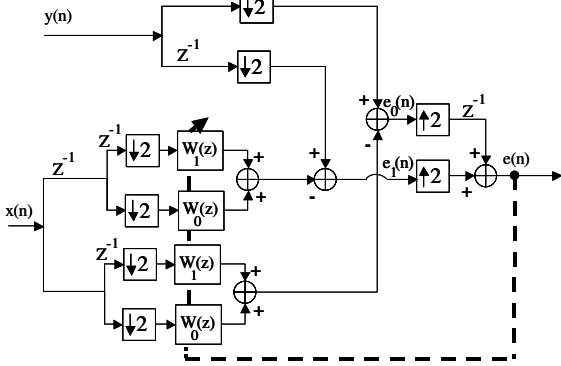Figure 4: Adaptive filter using a PR delay chain filterbank.

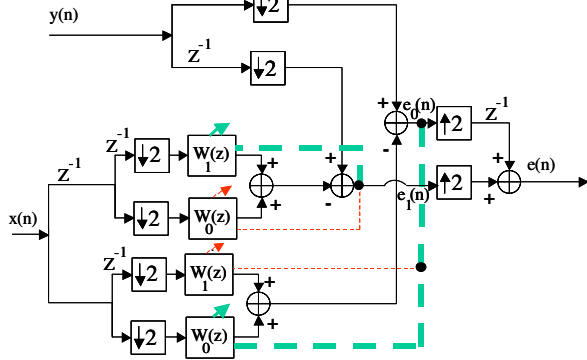Figure 5: PR delay chain with polyphase filters adapted jointly.

Figure 6: Polyphase adaptive filter representation using a delay chain. For S-NLMS, ignore the thin dashed lines

will constitute the subband adaptive filters. If all the D sub-filters are used, we obtain the S-NLMS modeled as a subband adaptive system employing a PR filterbank. In the proposed algorithm however, some of the polyphase components are zeroed-out, as depicted in Fig. 7 for $D = 4$ and $I = 2$.

We now analyze the introduced aliasing. In a PR delay chain filterbank, with decimation rate of D, output signal $y(n)$, is expressed in terms of input signal $x(n)$ by Eq. (4) [8],

$$Y(z) = \frac{z^{-(D-1)}}{D} \sum_{p=0}^{D-1} X(zW_D^p) \sum_{m=0}^{D-1} W_D^{-kp} \qquad (4)$$

where $W_D = e^{-j2\pi/D}$. The first summation (over p) in (4) simply shows how aliasing is created by addition of $X(z)$ and its
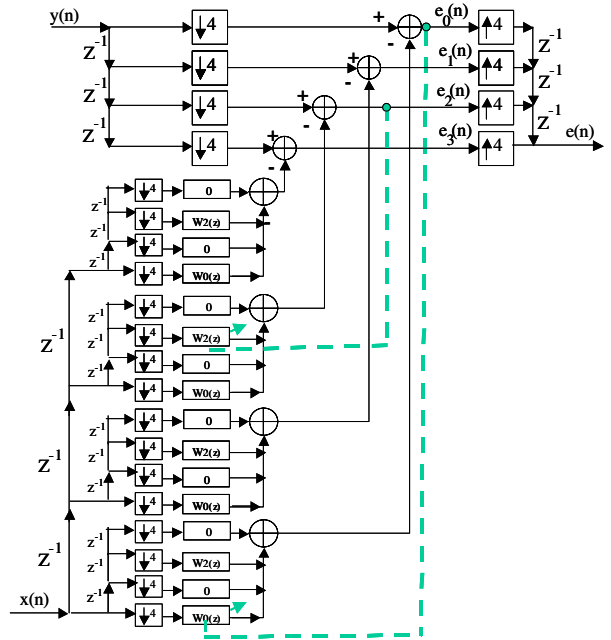

Figure 7. The proposed algorithm (with $D = 4, I = 2$) interpreted as S-NLMS with zeroed-out polyphase components.
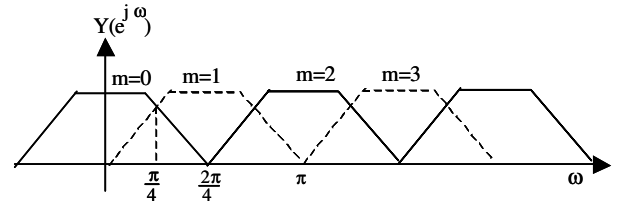

Figure 8: Subband spectrum of Equation (4) for the structure of Fig. 7, for $OS = 4$, $D = 4$, and $I = 2$.

modulated versions. The second summation (over m) is identically zero for $p > 0$, canceling the aliasing altogether. In our proposed algorithm however, only D/I of the D terms in the second sum are nonzero. As a result, aliasing will occur. Fortunately, subband signals (acting as $X(z)$ in (4)) in oversampled filterbanks are effectively bandlimited [3]. This seriously limits aliasing as long as $I \le OS/2$ for typical prototype filters in oversampled filterbanks. Fig. 8 depicts the output spectrum corresponding to the structure presented in Fig. 7. As shown, the spectral replicas represented by the dashed lines (corresponding to $m = 1, 3$ in Equation (4)) are eliminated, effectively removing the aliasing.

4. SIMULATION

The proposed algorithm was employed for echo cancellation using the OS-SAF system depicted in Fig. 1. In each subband complex subband signals $x_k(m)$ and $y_k(m)$ (see Fig. 1) were used as inputs to the proposed algorithm (in Fig. 3), generating a subband output signal of $e_k(m)$. Each subband adaptive filter consisted of 32 taps. The OS-SAF was configured for an analysis window length of 128 points, a synthesis window length of 32 points, an oversampling rate of $OS = 8$,
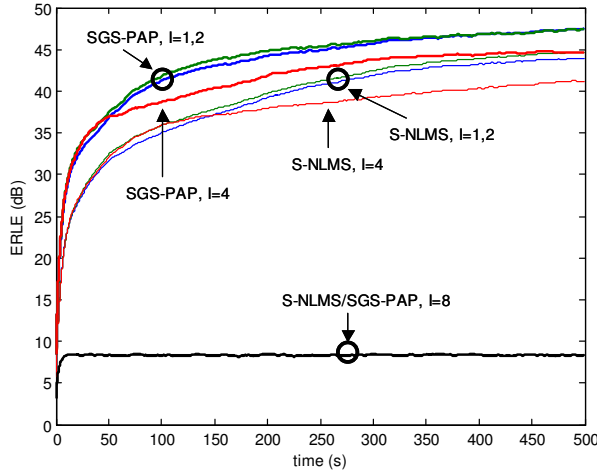
Figure 9. Fullband ERLE performance of the S-NLMS and SGS-PAP for $D=16$ and different imaging factors.

Table 1. Steady-State full-band ERLE (dB) for $D=16$ and different imaging factors.

I	S-NLMS ERLE	SGS-PAP ERLE
1	50.4	52.4
2	50.1	51.9
4	45.1	44.8
8	8.4	8.3

and 32 complex bands (of which 16 were unique because of Hermitian symmetry). It was necessary to keep the imaging factor, I, to at most $OS/2=4$ to limit the aliasing error as described in Section 3.

The reference signal used in simulation was white noise sampled at 8 kHz. The echo was generated using a typical acoustic plant, and normalized for an echo return loss of 10 dB. There was no near-end disturbance in the primary signal. The algorithm was applied to both the S-NLMS and the SGS-PAP (with affine order of two) described in [6].

Fig. 9 plots the fullband echo return loss enhancement (ERLE) against time, for the proposed algorithm with $D=16$ and various I's, and Table 1 lists the fullband steady-state ERLE's using the parameters employed for Fig. 9. As depicted in Fig. 9, considering the μ-scaling by I in (3), the initial convergence rate is almost the same for various I values. From both Fig. 9 and Table 1, the adverse effect of increasing I (particularly for $I=4$ and $I=8$) on the steady-state performance is evident. For $I=2$, the performances are almost identical to those for $I=1$, offering computation reductions of $I^2=4$ in adaptation and 2 times in filtering. For $I=4$, the steady-state ERLE drops to around 45 dB for S-NLMS and SGS-PAP. However, the computation cost reduction relative to S-NLMS (and SGS-PAP) is considerable: $I^2=16$ times reduction in adaptation and 4 times in filtering. Compared to WBD, the computation cost in adaptation has decreased by a factor of $D/I=4$. In reality, initial convergence is more crucial in tracking the plant. Moreover, practically the steady-state performance is often limited by the near-end disturbance that rarely allows ERLE values in excess of 40 dB. This implies that the observed performance reduction for

$I=4$ is not practically significant. Generally, performance variation trends of the S-NLMS and the SGS-PAP with different I values are similar. As expected, the SGS-PAP always outperforms the S-NLMS since the affine projection algorithm has superior convergence compared to NLMS.

The proposed algorithm was also simulated with the above reference signal using various D values. Convergence and steady-state performance were indistinguishable for fixed I. As mentioned in Section 2, similar to the results reported in [5,6] on sequential algorithms, this was expected due to μ-scaling by a factor of D in (3).

Finally, the proposed algorithm was simulated using a reference signal that consisted of speech and white background noise with a SNR of 25 dB. The echo was generated as above. The results were similar to the white noise case with no disparity worthy of report.

5.   CONCLUSIONS

In this paper, a method of computation cost reduction for partial update algorithms on OS-SAF platforms was proposed. The algorithm achieves a reduction in computation cost in sequential update algorithms while introducing minimal aliasing. In comparison with WBD, it is more flexible in terms of computation reduction and achieves this flexibility without sacrificing considerable performance for a stationary plant. The proposed algorithm is implemented on an ultra-low resource OS-SAF WOLA filterbank. It practically achieves a computation cost reduction of 16 times in adaptation and 4 times in filtering over S-NLMS, without practical performance reduction.

6.   REFERENCES

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